RDM Based Approach To Solving Decision Making Problem Under Uncertain Environment

Latafat Abbas Gardashova

Abstract— The combination of fuzzy logic tools and multi criteria decision making has a great relevance in the literature. Real life decision making problem under uncertainty is usually associated with information that may be incomplete or imprecise. The information on which decisions are based is uncertainty. Decision making theory is based on several type of uncertainty and tools. Computing with word (CwW) is very effective tool for decision making. Professor Zadeh the creator of CwW idea, formulated many challenge problems. Zadeh’s flight delay problem using relative-distance-measure is discussed in [1]. In this paper RDM is applied to solve real life decision making problem. A numerical example is used to illustrate the procedure of the presented approach.

Keywords – alternatives, decision making, portfolio selection, relative distance measure

I. INTRODUCTION

Decision making is constantly needed in the real world. In many situations, making choices depending on numerous factors limited to human ability is very difficult to deal with. Decision making is constantly needed in the real world. In a fuzzy environment, a standard method to express experts’ preferences usually uses numerical values assessed in an interval from zero to one. However, there are some difficult decisions where experts are not able to give exact numerical values to their preferences. In such cases, linguistic assessments are alternatively used instead of numerical values to express preference [1]-[3]. In [4] a new characterization of consistency based on the additive transitivity property of fuzzy preference relations has been proposed. This new characterization allows us to easily check the consistency between the experts’ opinions. Experts’ opinions are represented by using if-then rules. In [5] one of the knowledge discovery approach -linguistic summarization [6]-[8] approach to generate IF-then rules for causal databases is proposed. Both type-1 and interval type-2 fuzzy sets are considered. In a lot of technical literature [9]-[13] five-quality measures-the degree of truth, sufficient coverage, reliability, outlier and simplicity are defined. Among them the degree of reliability is especially valuable for finding the most reliable and representative rules.

In [14] Zadeh introduced the concept of Z-numbers to describe the uncertain information which is a more generalized notion. A Z-number is an ordered pair of fuzzy numbers (\(\tilde{A}, \tilde{B}\)). Here \(\tilde{A}\) is a value of some variable and \(\tilde{B}\) represents an idea of certainty or other closely related concept such as sureness, confidence, reliability, strength of truth, or probability. It should be noted that in everyday decision making most decisions are in the form of Z-numbers. Zadeh suggests some operations for computation with Z-numbers, using the extension principle. On Z-valuation using Zadeh’s Z-numbers problem is discussed in [15] by R. Yager. It was shown how to use these Z-numbers to provide information about an uncertain variable in the form of Z-valuations, assuming that this uncertain variable is random. Yager offers an illustration of a Z-valuation, showing how to make decisions and answer questions. It has also been discussed the relationship between Z-numbers and linguistic summaries [16]. Author of [15] has provided for a representation of Z-valuations in terms Dempster-Shafer belief structures [17], that made use of type-2 fuzzy sets.

In [18] the authors suggest an approach for converting a Z-number to a fuzzy number. The superiority of the given method is represented by its low computational difficulty, because of its application. From information structure view point converting Z-numbers to classical fuzzy numbers [19]-[20] conducts to loss of original information.

In literature researchers formulate the problem of decision making when probabilities of states of nature and outcomes of alternatives are described by Z-numbers. In these works the main disadvantage is related to the loss of information resulting from converting Z-numbers to fuzzy numbers.

At the moment the existing literature devoted to calculation and a reasoning with restrictions include well developed approaches and theories to deal with pure probabilistic restrictions. For calculation with probabilistic restrictions as probability distributions the known probabilistic arithmetic is used.
If information is presented in the fuzzy form, then fuzzy arithmetic is used. We are known, that fuzzy operations are based on interval algebra.

In [21] is given a number of limitations and the drawbacks of Moore’s interval arithmetic:
- the excess width effect;
- the dependency problem;
- difficulties with solving even simplest interval equations;
- problems with the of right-hand sides of the interval equations;
- absurd solutions and requests to introduce negative entropy into the system.

The alternative for Moore arithmetic[22]-[24] can be multidimensional RDM interval arithmetic.

Benefits of RDM arithmetic than interval arithmetic are described below[21]:
- complicated problems can be solved, thanks to the possibility of transforming equations;
- almost all laws of the of crisp numbers holds for RDM arithmetic;
- RDM arithmetic provides complete, multidimensional problem solutions from which various simplified representations such as cardinality distribution, a span of a solution (Moore’s solution) or a center of gravity can be derived.

The idea of multidimensional RDM arithmetic was developed by A. Piegat[25]-[28].

There are many methods and approaches to operate over fuzzy numbers.

Unfortunately, about one day there is no approach to calculation on Z-numbers.

In [29] a method of converting Z-number to classical fuzzy number has been discussed. Decision Making Using Z-numbers under Uncertain Environment is discussed in [30]. Authors for solving multi-criteria decision making problem using Z-numbers have converted Z-numbers to classical fuzzy number and a priority weight of each alternative is determined.

The paper is organized as follows. Section 2 discusses required definitions. The algorithm of proposed decision making method is presented in Section 3. Description of portfolio selection-real-life business problem and the experimental results of calculations are described in Section 4. The conclusion is presented in the fifth section.

II. PRELIMINARIES

Definition 1. A fuzzy set A is defined on a universe X may be given as:

\[ A = \{(x, \mu_A(x)) \mid x \in X\} \]

where \( \mu_A : X \rightarrow [0, 1] \) is the membership function. A membership value \( \mu_A(x) \) describes the degree of belongingness of \( x \in X \) in A.

Definition 2. Let \( \tilde{A} = (a_1, a_2, a_3) \) and \( \tilde{B} = (b_1, b_2, b_3) \) be two triangular fuzzy numbers. The graded mean integration representation of triangular fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) can be obtained [31], respectively, as follows:

\[
P(\tilde{A}) = \frac{1}{6}(a_1 + 4a_2 + a_3) \\
P(\tilde{B}) = \frac{1}{6}(b_1 + 4b_2 + b_3)
\]

The canonical representation of the multiplication operation on triangular fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) is defined as:

\[
P(\tilde{A} \odot \tilde{B}) = P(\tilde{A}) \times P(\tilde{B}) = \frac{1}{6}(a_1 + 4a_2 + a_3) \times \frac{1}{6}(b_1 + 4b_2 + b_3)
\]

Definition 3. A triangular fuzzy number \( \tilde{A} \) can be defined by a a triplet \((a_1, a_2, a_3)\), where the membership can be determined as the following equation:

\[
\mu_A(x) = \begin{cases} 
0, & x \in [-\infty, a_1] \\
\frac{x-a_1}{a_2-a_1}, & x \in [a_1, a_2] \\
\frac{a_3-x}{a_3-a_2}, & x \in [a_2, a_3] \\
0, & x \in [a_3, +\infty] 
\end{cases}
\]

Definition 4. A Z-number[14]

A Z-number is an ordered pair of fuzzy numbers, \((\tilde{A}, \tilde{B})\). \(\tilde{A}\) -is a fuzzy restriction on the values which a real-valued uncertain variable is allowed to take. \(\tilde{B}\) is a measure of reliability of the first component(Fig.1).

![Fig.1. A simple Z-number](Image)

Definition 5 [21]. Addition in RDM. Let X and Y are two intervals:

\[ X = [\bar{x}, \bar{y}] = \{x : x = x + \alpha_x (\bar{x} - \bar{y}), \alpha_x \in [0,1]\} \]

and

\[ Y = [\bar{y}, \bar{z}] = \{y : y = y + \alpha_y (\bar{y} - \bar{z}), \alpha_y \in [0,1]\} \]

\[ X + Y = \{x + y : x + y = \bar{x} + \alpha_x (\bar{x} - \bar{y}) + \bar{y} + \alpha_y (\bar{y} - \bar{z})\}
\]

\[ \alpha_x, \alpha_y \in [0,1] \]

Multiplication in RDM.

\[
X = [\bar{x}, \bar{y}] = \{x : x = x + \alpha_x (\bar{x} - \bar{y}), \alpha_x \in [0,1]\}
\]

and

\[ Y = [\bar{y}, \bar{z}] = \{y : y = y + \alpha_y (\bar{y} - \bar{z}), \alpha_y \in [0,1]\} \]

\[ X \cdot Y = \{x \cdot y : x \cdot y = (x + \alpha_x (\bar{x} - \bar{y})) \cdot (\bar{y} + \alpha_y (\bar{y} - \bar{z})\}
\]

\[ \alpha_x, \alpha_y \in [0,1] \]
Definition 6 [29]. The priority weight of each alternative can be defined as follows:

\[
priority = \sum w(Z_a)w(Z_f)
\]

where \(Z_a\) is the weight of the criteria, and \(Z_f\) is the value of each criteria.

III. ALGORITHM OF DECISION MAKING METHOD USING Z-NUMBERS

Algorithm of decision making method using Z-numbers under uncertain environment is as follows:
Step1. Construct the fuzzy decision making matrix for decision making problem.
Step 2. Transform the linguistic value to numerical value.
Step 3. Normalize the fuzzy decision making matrix.
Step 4. Convert the Z-numbers to crisp number.
Step 5. Convert the crisp number to the interval.
Step 6. Determine the priority weight of each alternative.

IV. PORTFOLIO SELECTION- REAL-LIFE BUSINESS PROBLEM

As it is known, portfolio selection problem is one of standard and most important problems in investment and financial research fields. Portfolio selection- real-life business problem is characterized by the following three conditions:
1. Economy declines
2. No change
3. Economy expands.

Let us give formal description of the portfolio selection problem as a problem of decision making with imprecise probabilities described as Z-numbers.

1). State of nature. States of nature are represented by the three economic conditions. During the portfolio selection it is very important to properly identify the economic condition. Taking this into account it is adequately to describe the economic condition by using Z-numbers. The set of states of nature \( S = \{s_1, s_2, s_3\} \) described Z-numbers are

Economy declines- \( s_1: \tilde{Z} (P(s_1), \tilde{B}_1) = \tilde{Z} (low, very high) \)
No change- \( s_2: \tilde{Z} (P(s_2), \tilde{B}_1) = \tilde{Z} (middle, very high) \)
Economy expands \( s_3: \tilde{Z} (P(s_3), \tilde{B}_1) = \tilde{Z} (very low, very high) \)

2). Alternatives. Alternatives are represented by the three acts (selection A,B,C).

3). Utilities. Utility of an alternatives for each state is described by Z-numbers. So, we will formulate portfolio selection method by using Z-numbers as a determination of the priority weight of each alternative.

Here we give an example portfolio selection- real-life business problem. An investor has a certain amount of money available to invest now. Three alternative portfolio selections are available. The estimated profits of each portfolio under each economic condition are indicated in the following payoff Table 1 by using Z-valuations.

<table>
<thead>
<tr>
<th>Table 1: Payoff table</th>
<th>Economy declines (Low,Very high)</th>
<th>No change (Middle,Very high)</th>
<th>Economy expands (Very low,Very high)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ((below norm),(Very middle low))</td>
<td>((Very middle norm),(Very middle))</td>
<td>((Very very high norm),(Very very high))</td>
<td></td>
</tr>
<tr>
<td>B ((very very low norm),(Very very low))</td>
<td>((low norm),(Low))</td>
<td>((Very very high norm),(Very very high))</td>
<td></td>
</tr>
<tr>
<td>C ((very very low norm),(Very low))</td>
<td>(norm),(Middle))</td>
<td>(high norm ), (High))</td>
<td></td>
</tr>
</tbody>
</table>

Let \( \tilde{A} \) be a fuzzy set of the universe of discourse \( X \) subjectively defined as follow:

\[
f_{\nu}(x) = \begin{cases} 
  x - 2.400, & -2.400 \leq x \leq -2.000 \\
  -1.600 - x, & -2.000 \leq x \leq -1.600 \\
  0.400, & -1.600 \leq x \leq -2.000 
\end{cases}
\]

\[
f_{\omega}(x) = \begin{cases} 
  x - 1.200, & -1.200 \leq x \leq -1.000 \\
  -0.800 - x, & -1.000 \leq x \leq -0.800 \\
  0.200, & -1.000 \leq x \leq -0.800 
\end{cases}
\]

\[
f_{\mu}(x) = \begin{cases} 
  x - 0.450, & 0.450 \leq x \leq 0.500 \\
  0.600 - x, & 0.500 \leq x \leq 0.600 \\
  0.100, & 0.600 \leq x \leq 1.000 
\end{cases}
\]

\[
f_{\nu}(x) = \begin{cases} 
  x - 0.900, & 0.900 \leq x \leq 1.000 \\
  2.000 - x, & 1.000 \leq x \leq 2.000 \\
  1.000, & 2.000 \leq x \leq 2.000 
\end{cases}
\]

\[
f_{\omega}(x) = \begin{cases} 
  x - 1.600, & 1.600 \leq x \leq 2.000 \\
  2.400 - x, & 2.000 \leq x \leq 2.400 \\
  0.400, & 2.400 \leq x \leq 2.400 
\end{cases}
\]

\[
f_{\mu}(x) = \begin{cases} 
  x - 4.000, & 4.000 \leq x \leq 5.000 \\
  6.000 - x, & 5.000 \leq x \leq 6.000 \\
  1.000, & 6.000 \leq x \leq 6.000 
\end{cases}
\]

\[
f_{\nu}(x) = \begin{cases} 
  x - 16.000, & 16.000 \leq x \leq 20.000 \\
  4.000, & 16.000 \leq x \leq 20.000 
\end{cases}
\]
Fuzzy restriction part of Z-numbers by Scale data is given as Table 2:

Table 2: Fuzzy restriction part of z-numbers

<table>
<thead>
<tr>
<th>Economy declines</th>
<th>No change</th>
<th>Economy expands</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0.2,0.3,0.4),VH)</td>
<td>(0.4,0.5,0.6),VH)</td>
<td>(0.1,0.2,0.3),VH)</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.45,0.5,0.6),VML)</td>
<td>((0.9,1.1,2),VM)</td>
<td>((1.8,2,2.4),M)</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.1,0.2,0.3),VL)</td>
<td>(1.6,2.2,4),M)</td>
<td>(4,5,6),H)</td>
</tr>
</tbody>
</table>

The problem is to determine portfolio selection. On the basis of his own past experience, the investor assigns the following probabilities to each economic condition:

\[ P(\text{Economy declines}) = \{\text{low; very high}\} = (0.2,0.3,0.4) \]

\[ P(\text{No change}) = \{\text{middle; very high}\} = (0.4,0.5,0.6) \]

\[ P(\text{Economy expands}) = \{\text{very low; very high}\} = (0.1,0.2,0.3) \]

There are three different choices, namely A,B,C. Take the three main criteria (Economy declines, No change, Economy expands) into consideration.

The linguistic criteria evaluation of the three criteria are given Table 3.

Table 3: Decision matrix with linguistic values

<table>
<thead>
<tr>
<th>Economy declines (LVH)</th>
<th>No change (M/VH)</th>
<th>Economy expands (VL/VH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0.45,0.5,0.6),VML)</td>
<td>(0.9,1.1,2),VM</td>
<td>(1.8,2,2.4),M</td>
</tr>
<tr>
<td>B</td>
<td>(1.6,2,2.4),M</td>
<td>(4,5,6),H</td>
</tr>
<tr>
<td>((0.2,0.1,0.2),VL)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to the membership function denoted by (1) the linguistic variable can be converted to numerical value, which is described as Table 4.

Table 4: Decision matrix with numerical values

<table>
<thead>
<tr>
<th>Economy declines (0.2,0.3,0.4), (0.8,1,1)</th>
<th>No change (0.4,0.5,0.6), (0.8,1,1)</th>
<th>Economy expands (0.1,0.2,0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.45,0.5,0.6), (0.26,0.27,0.28)</td>
<td>(0.9,1.1,2), (0.28,0.29,0.298)</td>
<td>(1.8,2,2.4), (0.32,0.33,0.38)</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.1,0.2,0.3), (0.16,0.18,0.2)</td>
<td>(1.6,2,2.4), (0.32,0.33,0.38)</td>
<td>(4,5,6), (0.42, 0.44, 0.46)</td>
</tr>
</tbody>
</table>

The normalized matrix result is described Table 5 and decision matrix which combines the restraint and reliability Table 6.

Table 5: The normalized matrix result

<table>
<thead>
<tr>
<th>Economy declines (0.2,0.3,0.4), (0.8,1,1)</th>
<th>No change (0.4,0.5,0.6), (0.8,1,1)</th>
<th>Economy expands (0.1,0.2,0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.2759,0.2778,0.281), (0.26,0.27,0.28)</td>
<td>(0.9,2950,0.296,0.307)</td>
<td>(0.32,0.33,0.38)</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.1,0.02,0.02), (0.16,0.18,0.2)</td>
<td>(0.31852, 0.333,0.3482)</td>
<td>(0.407,0.444,0.4818)</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.01,0.0222), (0.01,0.15))</td>
<td>(0.21481,0.222,0.2296)</td>
<td>(0.85785,0.88,0.81)</td>
</tr>
</tbody>
</table>

Table 6: Decision matrix which combines the restraint and reliability

<table>
<thead>
<tr>
<th>Economy declines (0.2,0.3,0.4), (0.8,1,1)</th>
<th>No change (0.4,0.5,0.6), (0.8,1,1)</th>
<th>Economy expands (0.1,0.2,0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.2759,0.2778,0.2818)</td>
<td>(0.295,0.296,0.3078)</td>
<td>(0.32,0.33,0.3818)</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0,0.0222), (0.16,0.18,0.2)</td>
<td>(0.21481,0.222,0.2296)</td>
<td>(0.85785,0.88,0.81)</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0.17037,0.18519,0.2)</td>
<td>(0.31852,0.333,0.34815)</td>
<td>(0.407,0.444,0.4818)</td>
</tr>
</tbody>
</table>

After normalizing the weight of criteria, we can get the final priority weight of the three Portfolio denoted by Table 8 which is getting by using Table 7.

Table 7: Decision matrix with crisp number

<table>
<thead>
<tr>
<th>Economy declines 0.29</th>
<th>No change 0.4833</th>
<th>Economy expands 0.19333</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.075083</td>
<td>0.086006</td>
</tr>
<tr>
<td>B</td>
<td>0.00034</td>
<td>0.048519</td>
</tr>
<tr>
<td>C</td>
<td>0.033333</td>
<td>0.112222</td>
</tr>
</tbody>
</table>

Table 8: The priority weight of the portfolio selection

<table>
<thead>
<tr>
<th>Economy declines (0.29,0.3)</th>
<th>No change (0.483,0.5)</th>
<th>Economy expands (0.19,0.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[0.0707,0.0808]</td>
<td>[0.1,0.11]</td>
</tr>
<tr>
<td>C</td>
<td>[0.000083,0.00085]</td>
<td>[0.9,0.942]</td>
</tr>
<tr>
<td>B</td>
<td>[0.033333,0.011122]</td>
<td>[0.19,0.195]</td>
</tr>
</tbody>
</table>

Priority weight for alternative A,B, and C is calculated by using addition and multiplication of RDM arithmetic.

Finally, we calculated the priority weight for each alternative and obtained the following results:

Priority(A) = 0.08805
Priority(B) = 0.212
Priority(C) = 0.105

So, the best alternative is B as one with the highest priority weight.

III. CONCLUSION

Z-number is a new notion proposed by Zadeh has more ability to describe the uncertain knowledge. In this paper, we solve the multi-criteria decision making using Z-number and RDM algebra and a method is proposed to deal with the Z-number. At last, a numerical example is used to illustrate to procedure of the proposed method.

REFERENCES


Latafat Abbas Gardashova has published over 155 research papers. Also he has published 7 books. 

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