Adaptive Generalized Predictive Control Applied to Motor Drive Axis

Mohamed Essahafi, Mustapha Ait Lafkih

Abstract—The topic of this article is the adaptive generalized predictive control (GPC) applied to the control of the speed of a digital axis. The system is used in CNC machine tools. Usually, the control of digital axes must obey quickly and effectively to changes in the input variable and continuously adapting with the machining conditions, and oppose the influence of external disturbances.

However, The Online Recursive Least Squares with Forgetting factor was adopted to estimate in real-time the system parameters and adjust instantaneously GPC controller parameters based on a modeling CARIMA (Controlled Auto-Regressive Integrated Moving Average).

The results of this control architecture have achieved the desired local performance with good rejection of load disturbances and a good robustness for the parametric variations due to operating point changes.

Index Terms—Generalized predictive control, adaptive control, RLS identification, CARIMA Model,

I. INTRODUCTION

Nowadays, cost productivity and control requirements encouraged manufacturers to develop new machines. Thus, the complexity of the structures of machine tools and machining process increased while the control strategies that are always based on simple regulations such as linear PID controllers (Proportional / Integral / Derivative). Indeed, this type of control makes it possible to achieve acceptable performance in an industrial context. Yet even if the PID control loops contain few parameters, adjustment of gains is not trivial and valid only for systems whose parameters are time-invariant and stationary. However in the machining area, the performances are formulated in terms of machining speed and position.

For all these reasons, it has been necessary to implement an adaptive control strategy to define a command that will ensure the desired performance to external disturbances or variations parametric system during machining operation. The synthesis of the generalized predictive control (GPC) proposed by Clarke [1], [2], is one of the most practiced in academic control methods. Also it has been used successfully in several applications industrial under different forms. To be used, it must be coupled with an online identification method [3], [4].

Manuscript received July 22, 2015.

Among the benefits of the generalized predictive control [5] indicate that it can be applied to processes has pure variable delay, and to point out useful robustness properties which are determined by the cost-function parameters and a pre-specified observer.

This paper proposes the application of the law of synthesis indirect adaptive generalized predictive control with a parametric identification by recursive least squares with forgetting factor (FFRLS).

The paper is organized as follows: the first part presents a theory of adaptive generalized predictive control based on identification FFRLS, Second, the description of the drive system of a digital axis. Simulation results of the predictive control AGPC are discussed in the third part.

II. ADAPTIVE GENERALIZED PREDICTIVE CONTROL (GPC)

A. Synthesis of Generalized Predictive control (GPC)

The method described in this paragraph is developed by Clarke [1], [2] given for the case of SISO models. The basic model, which is a discrete linear model CARIMA (Controlled Auto-Regressive Integrated Moving Average) defined to represent the dynamic behavior of a process around a nominal operating point that given by the following form:

$$A(q^{-1}).\Delta y(t) = B(q^{-1}).\Delta u(t) + e(t)$$
(1)

Where

t is the discrete time iteration expressed as an integer multiple of the sampling interval Te

- y(t) is the output vector at time t
- u(t) is the control vector at time t
- q^{-1} is the backward-shift operator $q^{-1}y(t+1)=y(t)$ $\Delta(q^{-1}) = 1-q^{-1}$ is the differential operator. $A(q^{-1})$, $B(q^{-1})$ are polynomials in q^{-1} .

That is:

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \dots + a_{na} q^{-na}$$
(2)

$$B(q^{-1}) = 1 + b_1 q^{-1} + b_2 q^{-2} + \dots + b_{nb} q^{-nb}$$
(3)

With $na = deg A(q^{-1})$ and $nb = deg B(q^{-1})$

 $e(t)\,is\,$ a sequence of uncorrelated random zero-mean sequence with finite variance σ^2 That is,

$$E[e(t)] = 0$$
 and $E[e(t).e(t)^T] = \sigma^2$

Mohamed Essahafi, Preparing a PhD thesis in adaptive control at the faculty of Sciences and Technology, University of Sultan Moulay Slimane, BéniMellal Morocco (email : mohamed.essahafi@yahoo.fr).

Mustapha AitLafkih, Professor at the Electrical Engineering Department, Faculty of Sciences and Technology, University of MoulaySlimane, BéniMellal Morocco(email : m.aitlfkih@fstbm.ac.ma)

We define an adaptive control as a set of techniques for automatic adjustment of the controllers in real time, in order to achieve or to maintain a desired level of performance of the control system when the parameters of the plant (disturbance) dynamic model are unknown and/or change in time as shown in Fig. 1





The criterion to be minimized is composed of the quadratic sum the output of prediction error and increments controls:

$$J = \sum_{N1}^{N2} er^2 + \lambda \sum_{1}^{Nu} (\Delta u(k+j-1))^2$$
(4)

With

 $er = w(k + j) - \hat{y}(k + j)$ $\Delta u_j(k + j) = 0, \text{ for } j < Nu$ w (k + j) of the reference values at the time k + jy (k + j) prediction of the output at time k + j

The (k + d - 1) future control increments at time k + j - 1, NI minimum prediction horizon and Nu control horizon, the increment of the prediction horizon, the horizon of N2 maximum prediction control weighting factor.

On a prediction horizon, the output is obtained by the following equation:

$$y(k+j/k) = Fj(q^{-1})y(k) + Hj(q^{-1})\Delta u(k-1) + Gj\Delta u(k+j-1) + Eje(k+j)$$
(5)

Where Fj, Ej, Gj, Hj are polynomials from solving diophantine equations.

The optimal prediction of the output y(k + j) in the least squares sense, given the information available at time this given by

$$y(k+j) = Gj(q^{-1})\Delta u(k+j-1) + lj$$
(6)

$$lj = Fj(q^{-1})y(k) + Hj(q^{-1})\Delta u(k-1)$$
(7)

After writing in matrix form we get:

$$Y = G\Delta U + L \tag{8}$$

With

$$\hat{Y}^{T} = [\hat{y}(k+1) \dots \hat{y}(k+N_{2})]$$

$$(9)$$

$$\Delta U^{*} = [\Delta u(k) \dots \Delta u(k+N_{2}-1)]$$
(10)
$$L^{T} = [l_{1}(k+1) \dots l_{N2}(k+N_{2})]$$
(11)

$$J = (W - Y^{T})(W - Y) + \lambda \Delta U^{T} \Delta U$$
(12)

After the substitution of upcoming events by their predictions based on the principle of certainty equivalence. The optimal ΔU_{opt} control vector that minimizes, without constraints, the criterion

$$\frac{\partial J(\Delta U)}{\partial \Delta U} = 0 \tag{13}$$

Is given by:

$$\Delta U_{opt} = [G^T G + \lambda I]^{-1} G^T (W - L)$$
⁽¹⁴⁾

In fact, only the first command is actually applied; so we deduce:

$$u_{opt}(k) = u(k-1) + \bar{G}(W-L)$$
(15)

With \overline{G} , the first row of the matrix $[G^T G + \lambda I]^{-1} G^T$

B. Parametric adaptation algorithm

The parametric adjustment as in Fig. 2 must be made to ensure the stability of the adaptive control system and possibly the required performance [6], [7]. Predictive control can be combined with a parametric adaptation algorithm for a robust adaptive control system through the principle of certainty equivalence of replacing the control model by its estimated each sampling period.



Fig 2: Scheme of Parametric adaptation algorithm

The self-adjusting controller is achieved by the real-time estimation of polynomial parameters A (q^{-1}) , B (q^{-1}) model by recursive identification algorithm following fixed forgetting factor:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{\varepsilon(k)\phi^{T}(k-1)F(k-1)}{1+\phi^{T}(k-1)F(k-1)\phi(k-1)}$$
(16)

$$\varepsilon(k) = y(k) - \hat{\theta}(k-1)\phi(k-1) \tag{17}$$

$$F(k) = \frac{1}{\eta} \left[F(k-1) - \frac{F(k-1)\phi(k-1)\phi^{T}(k-1)F(k-1)}{\eta + \phi^{T}(k-1)F(k-1)\phi(k-1)} \right]$$
(18)
With

$$\hat{\theta}(k) = \left[\hat{a}_1(k), \dots, \hat{b}_{nb}(k)\right]$$

$$\phi^T(k-1) =$$
(19)

$$[y(k-1), ..., y(k-na)), u(k-d), ..., u(k-nb)]$$
 (20)

Where $\hat{\theta}$ is the vector containing the coefficients of the polynomials A (q⁻¹), B (q⁻¹), ϕ measures variables vectors of F covariance matrix and η is the forgetting factor ($0 < \eta < 1$).

C. Control structure

The general architecture of a digital axis drive [8], [9] can be as follows, as in Fig 3



Fig 3: Structure of digital axis system Machine tool

D. Power Control Module "Chopper Bridge":

It is a reversible chopper voltage and current, it is based on a bridge structure carrying the switch K1 (D1, H1), K2(D1, H1), K3 (D1, H1) and K4 (D1, H1) by a PWM (*Pulse Width Modulation*) signal. Each Ki switch consists of a controlled unidirectional switch power transistors H, and a power diode D, it is through diodes we can achieve bidirectional switches.

The load current will flow well in both directions, this ensures the power reversibility of the chopper. The power transfer can occur in both directions (and the load receives the energy supplied). The operating point ride in the four quadrants of the diagram *uc*, *con*.

This allows, when feeding a continuous current motor to rotate in both directions and also to operate as a generator and, for example, of recovering electrical energy during braking.



Fig 4: chopper bridge with control interrupters

For operation in a given quadrant, two of the four switches remain blocked (even if they are sent boot commands), the other two operate simultaneously and are opened and closed together and periodically. (The order of the switches is of complementary type: T is the switching period of the chopper, and its duty cycle α sent in a PWM cycle.

The average value of uc(t) is: it can be calculated with the method of areas:

$$uc = ua(2\alpha - 1) \tag{21}$$

The average value is positive if the duty cycle is $\alpha > 0.5$ and negative if $\alpha < 0.5$. And it is between -Ua and + Ua.

Pulse Width Modulation (PWM) uses digital signals to control power applications, as well as being fairly easy to convert back to analog with a minimum of hardware.

E. DC Motor modelling

Motors, either linear or rotational, can be modelled by direct current motors, typical equations for direct current motors are given in (22), (23), (24), and (25). They are transformed in the discrete time domain to be implemented in the proposed model

$$E(t) = Ke.\,\Omega m(t) \tag{22}$$

$$U(t) = R.i(t) + L\frac{di(t)}{dt} + E(t)$$
(23)

$$Cm(t) = Kt.i(t) \tag{24}$$

$$Jeq\frac{d\Omega m(t)}{dt} = Cm(t) - Cr(t)$$
⁽²⁵⁾

E is the motor electromotive force, Ωm the instantaneous rotation velocity, *Ke* the constant of motor electromotive force. U is the voltage across armature terminals, *R* is the electric resistance, and *L* is the inductance, I the current. Cm is the motor torque, and *Kt* is the torque constant. *Jeq* is the equivalent inertia projected on the motor axis and *Cr* gathers all resistant torques.

F. Speed sensor

For the acquisition of the speed; we uses an incremental encoder of position for its simplicity of maneuver and the standard numerical output signal A and B.



Fig 5: sensor speed

G. PWM (Pulse-width modulation) signal

A PWM signal is an output of a micro-controller that allows, to control a chopper bridge. To fully control the process and keep the system a SISO process, the frequency of the signal is fixed and its duty cycle is variable.

Adaptive Generalized Predictive Control Applied to Motor Drive Axis



Fig 6: PWM signal

III. SIMULATIONS RESULTS:

To move and control the drive motor of the numerical axis we used the trapezoidal profile when the control is to provide initially a fixed acceleration phase followed by a linear movement in a constant speed and finally a constant deceleration. As in Fig. 7 the response of the response of the system to the desired speed profile described previously, and as we see the static error and tracking error are really minimal.

In order to illustrate the behavior of the above presented Adaptive Generalized Predictive Control algorithm, the simulation results of a numerical axis model obtained by using online identification technique as shown in Fig. 8. The model is chosen as follows:

$$y(t) = -2y(t-1) + 1, 1y(t-2) + q^{-4}[u(t) + 2u(t-1)] + e(t))$$
(26)

Regarding the estimation algorithm, the parameters of models have been initialized by zero vectors and covariance matrix $F(0) = 10^5$, with a forgetting factor fixe $\eta = 0.97$

For the GPC control algorithm, the horizon minimum prediction was set at a value N1 = 1, a horizon maximum prediction N2 = 14 and Nu = 7 for the horizon control.



Fig 7: Numerical axis output and control input with noise presence conditions: AGPC control



Fig 8: Tuned parameters of system model (CARIMA)

IV. CONCLUSION

In this publication we presented the synthesis of Adaptive generalized predictive control, Applied to the speed control of a digital axis machine tool. The choice of the generalized predictive control which has the advantage of being adaptively and extensible, allowed us to overcome the complexity of overall control of the system, while guaranteeing acceptable performance in pursuit and control with good robustness to changes of point operation that are causing by variations Parametric. The control of the position and speed of the axis uses a multi-variable modeling, which is to be discussed in another paper.

V. REFERENCES

- D.W. Clarke, C. Mohtadi and and P.S. Tuffs "Generalized predictive control- part I. The basic algorithm," Automatica, 23(2), pp. 137-148, 1987.
- [2] D.W. Clarke, C. Mohtadi and P.S. Tuffs "Generalized predictive control- part II. Extensions and interpretations," Automatica, 23(2), pp. 149-160, 1987.
 [3]
- [4] M. M'saad, J. Chebassier "Commande adaptative des systèmes, Techniques de l'ingénieur," vol. 7426, pp. 1–25, 2000.
- [5] R. Keohane (1958). Power and Interdependence: World Politics in Transition. Boston, Little, Brown & Co.
- [6] Z. Zidane, M. AitLafkih, M. Ramzi, "Adaptive Generalized Predictive Control of a Heat Exchanger Pilot Plant", Proceedings of the 2011 International conference on Multimedia Computing and Systems, April 7-9, 2011.
- [7] Adaptive Generalized Predictive Control of a Separately Excited DC Motor Mustapha AitLafkih, Zohra Zidane and Mohamed Ramzi
- [8] International Journal of Sciences and Techniques of Automatic control & computer engineering
- [9] IJ-STA, Volume 5, N° 1, June 2011, pp. 1472–1485.
- [10] Application of optimal adaptive generalized predictive control to a packed distillation column S Karacan, H Hapoğlu, MAlpbaz,
- [11] Department of Chemical Engineering, Faculty of Science, Ankara University, Tandoğan, Ankara, Turkey Available online 15 April 2002
- [12] Adaptive Prediction and Predictive Control—P. P. Kanjilal (IEE Control Engineering Series 52. London, U.K.: Peregrinus, 578 pp.)