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Classification of Some {0,1}-Semigraphs

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Abstract— **An adjacent graph is a connected bipartite {0,1}-semigraph which contains exactly one part in which any two vertices have exactly one common neighbour. Mulder [1] observed that; (0, λ) -semigraphs are regular. Furthermore a lower bound for the degree of (0,n)-semi graphs with diameter at least four was derived by Mulder [1]. In this paper, we find all -graphs and (0,1)-graphs. Furthermore, we determined some basic properties of adjacent graphs, where, λ≥1.**

Key words: **A-semigraph, bipartite graph**
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I. INTRODUCTION

Let us first recall some definitions and results. For more details, (see [1]). To facilitate the general definition of a graph, we first introduce the concept of the unordered product of a set V with itself.

Recall that the ordered product or cartesian product of a set V with itself, denoted by V×V, is defined to be the set of all ordered pairs (s; t), where s 2 V and t 2 V. Here (s; t) and (t; s) are considered to be distinct entities except when s = t. In a similar vein, the symbol {s,t} will denote an unordered pairs. A graph G=(V,E) consists of a finite nonempty set V of v vertices together with a prescribed set E of e unordered pairs of distinct vertices of V. Each pair u={x,y} of vertices in E is a edge of G and u is said to joins x and y. We write u = xy and say that vertices x and y are adjacent vertices; the vertex x and the edge u are incident with each other, as are y and u. If two distinct edges u and v are incident with a common vertex, then they are adjacent edges. A vertex z which adjacents to two distinct vertices x and y is called common neighbour of x and y. The neighborhood of a vertex x is the set N(x) consists of all vertices which are adjacent with x. The degree of a vertex p is the number d (p) of edges which are incident with it.

Let X be a subset of V. The integer n , where n + 1 = max {d(p) : p 2 Xg, is called the order of the set X. The minimum degree among the vertices of G=(V,E) is denoted by(G): If G=(V,E) contains a cycle, the girth of G=(V,E) denoted g(G) is the lenght of its shortest cycle.

Let G=(V,E) be a connected graph , X be a subset of V, A be a finite subset of non-negative integers and n (x; y) be the total number of common neighbours of any two vertices x; y of X.. The set X is called A-semiset if n (x; y) 2. for any two vertices x; y of X. If X is a A-semiset, but not B-semiset for any subset B of X ,the set X is called A-set. G=(V,E) is a A-semigraph (A-graph) if V is the A-semiset (A-set), respectively.

Mulder[1] observed that -semigraphs(⊥ ⊥ 2) are regular. Furthermore a lower bound for the degree of –semigraphs with diameter at least four was derived by Mulder [1]. In this paper, we …nd all f0g-graphs and -graphs. Furthermore we determined basic properties of some adjacent graphs.a {1}-set.

Definition 1.1 A bigraph (or bipartite graph) G=(P∪L,E) is a graph**.**

II. Main Results

Lemma 2.1. Let) G=(P∪L,E) be a bigraph with parts P and L. If the part P is a {1}-set, the part L is {0,1}-semiset and) G=(P∪L,E) is adjacent or biadjacent.

Proof 2.1. Let) G=(P∪L,E) be a bigraph with parts P and L and the part P be a {1}-set.

Assume that the part L does not {0,1}-semiset. Then the part L has at least two distinct vertices u and w having at least two distinct common neighbours x and y in the part P. This contradict to chosen of the part P. Thus the part L is {0,1}-semiset and) G=(P∪L,E) is adjacented or biadjacent.

Lemma 2.2. The intersection of any number of convex subgraphs of a graph G=(V,E) is a convex subgraph.

Proof 2.2. Let X_i be subset of V and X be the intersection of any number of convex graphs $G_i = (X_i; E_i)$ on X_i for any nonnegative integer i.

We need only show that, if p and q are vertices of X and the vertices p and q have common neighbour r, N(r)↔ X_i for each i. But, any convex graph containing X contains the vertices p and q, and so by de…nition the neighborhood N(r). Therefore, N(r) is in all convexs graphs of which X is the intersection, and so N(r) → X.

Proposition 2.1 Let X be any set of vertices of a graph G. A convex subgraph which contains X, but does not properly contain any convex subgraph which contains X is called the closure of X denoted by [X].

It is not obvious from the definition that the closure of X is a unique, but this follows lemma below. Thus, the closure of X is the smallest convex graph containinig of X. It is clear that [in any graph G=(V,E). Also, for any subset X of V, X [X], [[X]] = [X] and if X Y then [X] [Y]

Lemma 2.3. The closure of any subset X of a graph G is the intersection of all convex graphs on X.

Proof 2.3. By lemma2.1, this intersection is a convex subgraph of G. It is Definition 2.1 Let X be any set of vertices of a graph G. If for each vertex x of X x =2 [X], the set X is called independent. A basis of a G=(V,E) is an independent subset of V which generates V.

It is not obvious from the definition that a basis of a partial adjacented bigraph G=(V,E) is not necessarily unique. The

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incidence graph of Fano plane is a adjacented bigaph having many more diferent bases. For a given partial adjacented bigraph, do all bases have the same number of elements?

The answer is no, as can be seen by considering the example 2.1. above.

Example 2.1. Let P={p1; p2; p3; p4; p5; p6; p7; p8; p9}; L={l1; l2; l3; l4; l5; l6; l7; l8; l9}, P \ L=;, and N(l1) = {p1; p2; p3}; N(l2) = {p1; p4; p5}; N(l3) = {p3; p5}; N(l4) = {p6; p7}; N(l5) = {p7; p8}, N(l6) = {p6; p9}; N(l7) = {p8; p9}; N(l8) = {p2; p5; p7; p9}; N(l8) = {p3; p4; p6; p8}. Then the set P [L determines a partial adjacented bigraph) G=(P∪L,E). In this graph, the set {p1; p3; p5} is a basis while so is the set {p6; p7; p8; p9}.

Definition 2.2. Let G=(P∪L,E) be a bigraph with parts P and L. and) G=(P∪L,E) the part P be {1}-set |P|= v , |L| = b; the vertices of P will be labelled p1, p2,…, pv. Similary the vertices of L will be labelled ℓ1; ℓ2,… ℓb: To make our notation even more concise we define, wi = a(ℓi) : The total number vertices which are adjacent to the vertex ℓi bi = a(pi): The total number vertices which are adjacent to the vertex pi.

Definition 2.3. If pi∈ℓj, rij=1,and if pi∉ℓj, rij=0

Proof 2.4. If we add the 1's in each column, column by column, we get If we add the 1 s in each row, row by row, we get bi. But obviously we are just counting the same number of 1's in two diferent ways so we have the equations.

Definition 2.4. Let G=(PỤL,E) be a bigraph with parts P and L. and the part P be (semiadjacent) adjacent Let pi∈P, ℓj∈L and pi∈ℓj. The total number of paths which are between pi and lj is called the path number denoted p(pi,lj)=pij. If pi∈lj, pij=1. Hence pij =1 if rij = 1:

Lemma 2.5. Let G=(P ỤL,E) be a bigraph with parts P and L, and the part P be (semiadjacent) adjacent and for any vertex pi of P and vertex lj of L, pi∈lj, pij ¶ bi = a(pi) and d(pi; lj) = 3 or d(pi; lj) = 1.

Proof. This follows easily from if pi∈ℓj, rij = 1 it follows from semiadjacent part P.

Lemma 1.6. If rij = 0 then the number adjacent vertices to pi and don't have common neighbour to lj is a(pi)- pij .

Proof : Since a(pi) is the total number of vertices which are adjacent with pi and by definition pij, the result is immediate.

Proposition 1.7. If G = (BỤW;E) is a (weakly adjacent) bigraph with parts B and W, B is weakly adjacent part of G and pij = a(lj) for every vertex pi of B and vertex pi. of W such that rij = 0 then B is a part bigraph.

Proof.. Since jWj = b 1; there is a vertex pk of W, say. We must show that the set B is adjacent, that is, for any distinct two vertices pi and pj of B, a(pi; pj)=1. Let pi; pj be two distinct vertices of B. If rik = rjk =1, a(pi; pj) =1. If rik = 0 and rjk = 1 then by assumption pik = a(ℓk) so that it is easy to see that it is the smallest convex graph on X as any convex graph on X is included when we take the intersection.We say that X generates its closure.

Conversely, given a convex subgraph G0, we say that X is a generating set for G0 if [X] = G0, so that also X generates G0 has a common neighbour with vertex which is adjacent to lk. In particular, pi and pj have common neighbour. Thus a(pi; pj) = 1 Finally If rik = rjk = 0, using the hypothesis once again, for a vertex q which is adjacent with lk a(pi; q) = 1. If the vertex pj is adjacent with common neighbour of vertices pi and q, a(pi; pj)=1 and otherwise, apply the hypothesis one last time to get a common neighbour of vertices pi and pj . Therefore a(pi; pj) = 1, that is, G is adjacent.

Proposition 1.8. Let G = (B ỤW;E) be a(weakly adjacent) bigraph with v+b vertices and parts B and W, |B| = v,.|W| = b and B is weakly adjacent part of G. Then if B is a adjacent part, Σ vj(vj-1) = v(v-1).

Proof. Suppose that G is a adjacent bigraph. Then B is a adjacent part of G. We count the number of pairs of vertices of B in two di¤erent ways. First of all, there are v 2 pairs of vertices (counting {pi,pj} to be the same pair as {pj,pi}) or v(v-1) 2 .

Proposition 1.9. Let G=(V;E) be a graph, P be a maximal nonadjacent vertex set of V and L={Vl ,V j N(ℓ)↔P, |N(ℓ)| and p P, l 2 L, p 2 l , p 2 N(l). If P is a (weakly) adjacent subset of V , the structure S = (PỤL,E) is a (near) linear space.

Proof. Each vertex ℓ of L is common neighbour of at least two distinct vertices x and y of P, since |N(ℓ)|=2. Therefore v() = jlj ⊥ 2. Thus NL1 holds. For two distinct vertices a nad y of Þ, since a(x; y) ¶ 1 the vertices(points) x and y have at most one common neighbour(line). Thus NL2 holds in S. Therefore, S = (P; L;E) is a (near) linear space.

Corollary. If G = (Þ[L;E) is (weakly) adjacent bigraph with(weakly) adjacent part Þ, (Þ; L0;E) is a (near)linear space where L0 = fl 2 L j N(l) → Þ , jN(l)j ⊥ 2g Let G(Þ,L;E) denote to the graph with parts Þ and L. The point p 2 Þ lies on the line l 2 L if the vertex p is adjacent to the vertex l in G.

Proposition 1.10 Let G = (Pv [Lb; E) be a bigraph with parts Þ and L, the part Þ of G be a weakly adjacent set and ordered pairs of vertices adjacent to a vertex pi of P. So the left hand side of the inequality counts the number of ordered 6 pairs of coadjacent vertices of L. Clearly there are altogether |L|(|L|-1) ordered pairs of vertices of L. Thus the equality holds.

Proposition 1.11. Let G be a weakly adjacent bigraph with parts Þ and L, the set Þ be a weakly adjacent set.

The set L of G is adjacent part of G if X p∈P , a(pi)(a(pi)-1)= |L| (|L|-1)

Proof It is clear from proposition 1.10.

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