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Fuzzy Modified TOPSIS for Supplier Selection Problem in Supply Chain Management

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Abstract— **Nowadays, global market is highly competitive. Major part of the capital is spent on purchasing raw material/semi finished items. The strategic decision of supply chain is to minimize the expenses on the purchase of items. There are several criteria involved in this problem; such as cost, quality, on-time delivery and long term relationship. All of these criteria are conflicting in nature and linguistic that is why technique for order preference to positive ideal solution (TOPSIS) is used. Modified TOPSIS method is invented to deal with such problems. Modified TOPSIS method is fusion of TOPSIS method and linear programming problem (LPP) method along with more variable transform $sin(x^*)$. In this method, $sin(x^*),\ x^* \in \left[0, \frac{\pi}{2}\right]$ transform is used for normalization. LPP method is used to maximize the closeness coefficient for obtaining the optimal order quantity. The modified TOPSIS method not only ranks the supplier but also it gives us the idea about how much to order from the selected supplier. Thumb of rule is that more the variable transforms better the closeness coefficient. The numerical example is given to illustrate the above methods.**

*Keywords— **Supply Chain; Supplier Selection; Fuzzy; Modified TOPSIS; TOPSIS Method.***

I. INTRODUCTION

A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers. Supply chain exists in both service and manufacturing organizations, although the complexity of the chain may vary greatly from industry to industry and firm to firm. Suppliers are a part of a well-managed and established supply chain. The supplier selection problem becomes one of the most important issues for establishing effective supply chain.

The environment in global market is uncertain. While selecting the best suppliers are the splendor challenge in front of management team. Best suppliers mean those suppliers who supply the raw material / semi-finished items with lower cost and higher quality. Items should deliver immediately as per requirement of manufacturing firm. In the present study, a long term relationship new criterion is included.

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Manufacturing firm selects the suppliers by considering above three criteria other than long term relationship. Suppliers send material to manufacturing firm. After sometime, if these suppliers stop to send material to the firm then there is the problem to the manufacturing firm such as production line is interrupted.

Ultimately there is direct financial loss that is why long term relationship criteria is included in this model. Sometimes, manufacturer selects suppliers only on job work basis. All the machineries and dies of manufacturing firm are set up in the suppliers firm. In this kind of deal, if suppliers stop to send material to manufacturer then there is also the loss to the manufacturer. Loss due to the cost incurred to set up dies and machineries. Since, long term relationship is the best criteria in this model. During 1990s, many manufacturers seek to collaborate with their suppliers in order to upgrade their management performance and competitiveness [4]. The flow of material is shown in Fig.1.

In a real situation for a multi-objective supplier selection problem (MOSS), previous information is not exactly known. We use fuzzy set theory. Deterministic model cannot easily take this vagueness into account. In these cases the theory of fuzzy sets is one of the best tools for handling uncertainty. Fuzzy set theories are employed due to the presence of vagueness and imprecision of information in the supplier selection problem. Zimmerman (1978) and Zimmerman (1987) first used the Bellman and Zadeh (1970) method to solve fuzzy goals and fuzzy constraints are treated equivalently. Dulmin et al. (2003) studied the vendor selection model. Relationship between suppliers and manufacturing plants is very important for effective management of supply chain. Some academician studied on multiple attribute decision making criteria. Chen et al. (2005) developed supplier selection model. Success of supply chain management is depends on its suppliers Choi et al. (1996).

This model can be used as a decision support system by the decision maker (DM) to select the suppliers and decide what order quantity to place with each supplier in case of multiple sourcing of single item. In a real situation, for a supplier selection problem, most of the information is not known precisely. It is very difficult to take decision. Such vague terms are “very good in quality”, “very poor in on time delivery” and “medium in cost”. Deterministic models cannot take this vagueness into account. The ratings of qualitative criteria are linguistic in nature. It is a multiple criteria decision making problem. Usually, some of criteria are conflicting in nature. Therefore, it is reasonable to use fuzzy set theory and Dempster Shafer theory of evidence (DST) (Deng et al. 2011). Here, the main idea of the technique for order preference by similarity to an ideal solution (TOPSIS), is developed to deal with candidate selection problem [Chen, 2000]. TOPSIS is a multiple criteria method to identify solutions from a finite set of alternatives based upon simultaneous minimization of distance from an ideal point and maximization of distance from negative ideal point. TOPSIS can incorporate relative weights of criterion importance [Olson, 2004]. The fuzzy modified TOPSIS method is a fusion of usual linguistic approach and quantitative approach. Fuzzy TOPSIS method gives idea about the ranking of supplier but it does not convey the how much order to from suppliers. There is no such method in the literature. This method works as like a dual process. Qualitative input is given to the first process to get quantitative output and output of the first process and some additional information treated as input of second process to get the final output. In order to develop the modified TOPSIS method, $sin(x^*),\ x^* \in \left[0, \frac{\pi}{2}\right]$ normalization transform is applied and closeness coefficient is maximized subject to the constraints. Modified TOPSIS method is fusion of fuzzy TOPSIS method and linear programming problem (LPP) method along with more variable transform $sin(x^*)$. Thumb of rule is that more the variable transforms better the closeness coefficient. $sin(x^*)$ transform is more variable as compared to transform used for normalization in fuzzy TOPSIS method.

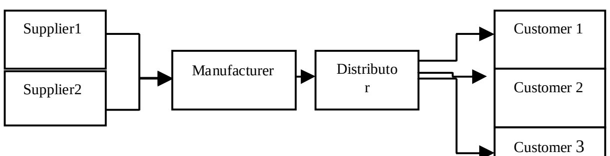

Fig.1. Flow of material in supply chain

The paper is organized as follows. Section 2 includes the basic definition and notations of fuzzy number and linguistic variable. Section 3 describes the proposed modified TOPSIS method. The proposed method is illustrated with an example in section4 followed by results and discussion are given in section 5. At the end conclusions are given in section 6.

II. PRELIMINARIES

Following is some definitions and notations that are used throughout the discussion [Kauffman et al. (1985) & Zimmermann (1991)].

2.1.1. Fuzzy set theory

Definition 2.1. In fuzzy sets, each element is mapped to [0,1] by membership function.

$$\mu_{\tilde{A}}(x): X \to [0,1]$$

where [0, 1] means real numbers between 0 and 1.

Definition2.2. A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ $\forall\ x_1, x_2 \in X,\ \lambda \in [0,1]$ .Where min denotes minimum operator (Klir et al. 1995).

Definition 2.3. A fuzzy $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that $\exists x_i \in X,\ \mu_{\tilde{A}}(x_i) = 1$.

Definition 2.4. If a fuzzy set is convex and normalized, and its membership function is defined in $\Re$ and piecewise continuous, it is called as fuzzy number.

Definition 2.5. Union of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{A} \cup \tilde{B}$ is defined as that fuzzy set on X for which

$$\mu_{\tilde{A}\cup\tilde{B}}(x) = \max[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] \qquad \forall x \in X.$$

Definition 2.6. Intersection of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{A} \cap \tilde{B}$ is defined as that fuzzy set on X for which

$$\mu_{\tilde{A}\cap\tilde{B}}(x) = \min[\mu_{\tilde{A}}(x),\ \mu_{\tilde{B}}(x)] \qquad \forall x \in X.$$

Definition 2.7. It is a fuzzy number represented with three points as follows:

$\tilde{n} = (n_1, n_2, n_3)$. This representation is interpreted as membership functions and holds the following conditions,

i. n_1 to n_2 is increasing function.
ii. n_2 to n_3 is decreasing function.
iii. $n_1 \leq n_2 \leq n_3$.

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, & x < n_1 \\ \dfrac{x - n_1}{n_2 - n_1}, & n_1 \leq x < n_2 \\ \dfrac{n_3 - x}{n_3 - n_2}, & n_2 \leq x \leq n_3 \\ 0, & x > n_3 \end{cases}$$

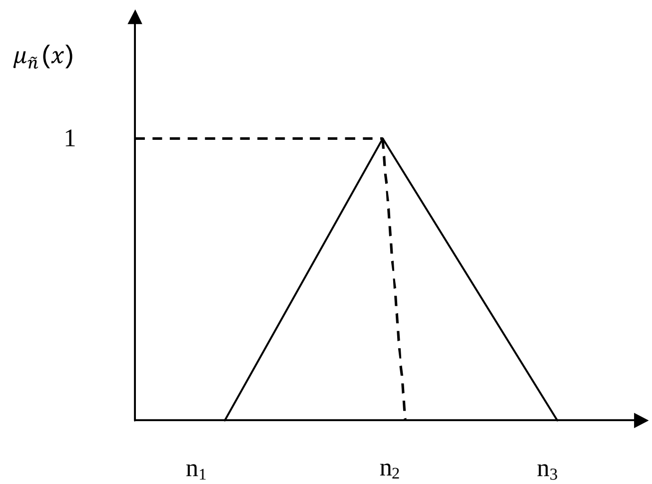

Fig. 2 Triangular fuzzy number

2.8. Euclidean Distance:

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}$$

Definition 2.9. Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers. The fuzzy number $\tilde{A}$ is closer to fuzzy number $\tilde{B}$ as $d(\tilde{A}, \tilde{B})$ approaches 0.

2.1.2. Defuzzification:

Defuzzification is an important step in fuzzy modeling and fuzzy multi-criteria decision-making. The defuzzification entails converting the fuzzy value into a crisp value and determining the ordinal positions of n-fuzzy parameters vector. Many defuzzification technique are available [Zimmermann (1991)], but the common defuzzification methods include centre of area, first of maximums, last of maximums and middle of maximums [Deng et al. (2011)].

Different defuzzification techniques extract different levels of information. In this paper, the canonical representation of operation on triangular fuzzy numbers [Deng et al. (2011)]. Which is based on the graded mean integration representation method is used in defuzziness process [Deng et al. (2011)].

Definition 2.10. Given a triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ the graded mean integration representation of triangular fuzzy number $\tilde{A}$ is defined as

.

$$P(\tilde{A}) = \frac{1}{6}(a_1 + 4a_2 + a_3) \quad (1)$$

By applying the equation (1), the graded mean integration representation for importance weight of cost criterion and ratings are shown in Table (15).

III. MODIFIED TOPSIS METHOD

TOPSIS method is developed by Hwang et al.(1981). In decision making process, the data is vague, the crisp value are inadequate to model real world problem. In this case, the ratings of each alternative and the weight of each criterion are described by linguistic terms which can be expressed in triangular fuzzy numbers. Using the concept of fuzzy TOPSIS, a closeness coefficient is calculated. It is ratio distance from fuzzy negative ideal solution to sum of fuzzy negative ideal solution and fuzzy positive ideal solution. Here, our objective is to find best suppliers among the set of all available suppliers [Chen (2000)]. A multi-criteria decision making (MCDM) problem can be expressed in matrix form as

$$\tilde{D} = \begin{array}{c} \\ S_1 \\ S_2 \\ \cdot \\ \cdot \\ \cdot \\ S_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{array}$$

$\widetilde{W} = [\widetilde{w}_1, \widetilde{w}_2, \ldots, \widetilde{w}_n]$,

Where $S_1, S_2, \ldots, S_m$ are suppliers, $C_1, C_2, \ldots C_n$ are criteria with which suppliers performance are measured, $\tilde{x}_{ij}$ is the rating of suppliers S_i with respect to criterion C_j, $\widetilde{w}_j$ is the weight of criterion C_j. This method is proposed to solve fuzzy group decision making problem.

Table 1: Linguistic Variables for the importance of each Criterion

Linguistic Variables	Ratings
Very Low (VL)	(0, 0.1, 0.2)
Low (L)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
High (H)	(0.6, 0.8, 0.9)
Very High (VH)	(0.8, 0.9, 1)

Table 2: Linguistic Variables for the ratings

Linguistic Variables	Ratings
Very Poor (VP)	(0, 1, 2)
Poor (P)	(1, 3, 5)
Medium (M)	(3, 5, 7)
Good (G)	(6, 8, 9)
Very good (VG)	(8, 9, 10)

The importance of weights and the ratings of qualitative criteria are considered as linguistic variables. It can be expressed in positive triangular fuzzy numbers as shown in above Table 1 and 2. Let the fuzzy ratings of k^{th} Decision Maker (DM) is $\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$ where $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$, respectively. Therefore, the aggregated fuzzy ratings, $\tilde{x}_{ijk}$, of suppliers with respect to each criterion can be calculated according to [4]:

$$\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$$

where

$$a_{ij} = \min_k\{a_{ijk}\}, \qquad b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk},$$

$$c_{ij} = \max_k\{c_{ijk}\}$$

And the importance weight of criteria can be aggregated as

$$\widetilde{w}_j = \frac{1}{k}[\widetilde{w}_j^1(+)\widetilde{w}_j^2(+)\ldots(+)\widetilde{w}_j^k]$$

As stated above, supplier selection problem can be expressed in a matrix as follows [Chen (2000)& Chen et al. (2005)]:

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix},$$

$\widetilde{W} = [\widetilde{w}_1, \widetilde{w}_2, \ldots \widetilde{w}_n]$,

The real value of $\frac{\pi}{2}$ is 1.57. The fuzzy triangular numbers can be expressed as in [0, 1.57] form by multiplying 1.57 to the triangular fuzzy numbers. In this method $sin(x^*)$, $x^* \in \left[0, \frac{\pi}{2}\right]$ normalization transform is used.

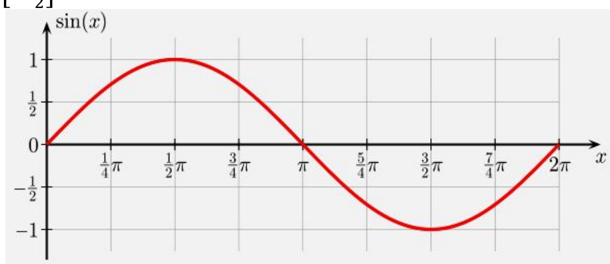

Fig.3 sin(x) function

Fig.3 clearly shows that the $sin(x^*)$ is a normalization transform only for the range $[0, \frac{\pi}{2}]$.

The normalized fuzzy decision matrix can be expressed as:

$$\tilde{R} = [\tilde{r}_{ij}], \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n$$

Where the $\tilde{r}_{ij}$ is the normalised value of $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$, which can be evaluated as $sin(\tilde{x}_{ij})$. The value of $\sin\left(\frac{\pi}{2}\right) = 1$ which is consistent with definition 2.3 of normalization.

A weighted normalized fuzzy decision matrix can obtained as follows using definition 2.9:

$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}$,

where $\tilde{v}_{ij} = \tilde{r}_{ij}(.)\widetilde{w}_j$.

Define the fuzzy positive-ideal solution (FPIS), S^* and fuzzy negative ideal solution (FNIS), S^- as

$$S^* = (\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*)$$

$$S^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-)$$

where $\tilde{v}_j^* = (1, 1, 1)$ and $\tilde{v}_j^- = (0, 0, 0)$

The distance of each supplier is calculated [Chen 2000].

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1, 2, \ldots, m,$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1, 2, \ldots, m,$$

where $d(.,.)$ is the distance measurement between two fuzzy numbers.

A closeness coefficient is defined to determine the ranking order of all suppliers once the d_i^* and d_i^- of each supplier S_i $(i = 1, 2, \ldots, m)$ has been calculated. The closeness coefficient of each supplier is calculated as

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \quad , i = 1, 2, \ldots, m.$$

CC_i approaches to 1 then supplier S_i is closer to the FPIS. Hence, using the closeness coefficient, we can obtain the ranking order of all suppliers and select the best one among a set of suppliers.

This method is an integration of DM's preference for supplier selection criteria subject to some constraints; for example, suppliers capacity, budget and demand of manufacturing firm. Using LPP method, we get order quantities from each suppliers. TOPSIS is a technique to rank several alternatives and LPP is used for resource distribution. These two methods are concatenated by maximizing the closeness coefficient subject to some constraints. This is the beauty of Modified TOPSIS method.

The algorithm of the multi-person multi-criteria decision making with fuzzy modified TOPSIS and LPP method for dealing with the supplier selection is given as follows:

Step1: Select a group of decision makers and identify the evaluation criteria.

Step2: Choose the appropriate linguistic variables for importance weight of criteria and linguistic ratings for suppliers.

Step3: Construct the fuzzy decision matrix and use $sin(x^*)$ transform to normalize the fuzzy decision matrix.

Step4: Obtain weighted normalized fuzzy decision matrix.

Step5: Determine FPIS and FNIS.

Step6: Calculate the distance of each supplier from FPIS and FNIS, respectively.

Step7: Evaluate the closeness coefficient of each supplier.

Step8: Using the closeness coefficient, obtain ranking of suppliers.

Step 9: Formulate the LPP for closeness coefficient (maximization) subject constraints.

Step 10: Solve this LPP to get the optimal order quantity for selected suppliers.

In order to formulate the model, the following notations are defined. The final model which integrates fuzzy modified TOPSIS and LPP can be shown as:

m number of supplier.

CC_i closeness coefficient of i[th] supplier.

D demand over period

p_i capacity of i[th] supplier to supply the item.

B_i total available budget of i[th] supplier.

Objective function of closeness coefficient:

$$Maximize\ Z = \sum_{i=1}^{m} CC_i x_i \qquad (2)$$

Subject to constraint

$$\sum_{i=1}^{m} x_i = D \qquad (3)$$

Constraint (2) is due to aggregate demand of item.

$$x_i \le p_i \quad , \quad i = 1, 2, \ldots, m. \qquad (4)$$

Constraint (3) is due to the maximum capacity of the suppliers.

$$c_i x_i \le B_i \quad , \quad i = 1, 2, \ldots, m. \qquad (5)$$

Constraint (4) is due to budget allocated to the suppliers.

$$x_i \ge 0\ and\ integer \quad , \quad i = 1, 2, \ldots, m. \quad (6)$$

Constraint (5) is nonnegative restriction and all order quantities are integers.

Where c_i is the cost coefficient. Which is obtained by using graded mean integration method (equation 1).

IV. NUMERICAL EXAMPLE

An automobile manufacturing company desires to select raw material suppliers. After the screening, five suppliers (S_1, S_2, S_3, S_4, S_5) left for evaluation. A group of decision-makers, D_1, D_2 and D_3, has been formed to select most suitable supplier. Four criteria are considered:

(1) Cost of a supplied item (C_1).
(2) Quality of a supplied item (C_2).
(3) On time delivery of a supplied item (C_3).
(4) Long term relationship with supplier (C_4).

After, examining the suppliers following information is collected. Demand over period is 2000.

Table 3: Suppliers budget and capacity

Suppliers	Capacity (p_i)	Budget in $ (B_i)
S_1	800	6,000
S_2	700	5,000
S_3	750	6,000
S_4	850	6,000
S_5	650	5,000

Table 4: The importance weight of criteria

	D_1	D_2	D_3
C_1	VH	H	H
C_2	M	H	M
C_3	H	M	H
C_4	H	H	VH

Table 5: Data for national level supplier selection:

Decision Makers	Performance	C_1	C_2	C_3	C_4
D_1	S_1	G	M	VG	G
	S_2	P	G	VG	M
	S_3	P	VP	P	G
	S_4	P	P	G	M
	S_5	VG	VG	VG	G
D_2	S_1	G	VG	G	G
	S_2	P	G	M	M
	S_3	P	M	P	G
	S_4	P	P	M	M
	S_5	VG	VG	M	G
D_3	S_1	G	VG	M	VG
	S_2	P	G	VG	G
	S_3	P	P	VP	M
	S_4	P	M	G	P
	S_5	VG	VG	VG	VG

Model algorithm is used to solve this numerical example. Convert the linguistic evaluation into the triangular fuzzy numbers and calculate the aggregated fuzzy rating. We get the fuzzy decision matrix as follows:

Table 6: Fuzzy-decision matrix and fuzzy weights of five candidates

Performance	C_1	C_2	C_3	C_4
Weight	[0.667, 0.833, 0.933]	[0.4, 0.6, 0.767]	[0.4, 0.6, 0.767]	[0.667, 0.833, 0.933]
S_1	[6, 8, 9]	[3, 7.67, 10]	[3, 7.33, 10]	[6, 8.67, 10]
S_2	[1, 3, 5]	[6, 8, 9]	[3, 7.67, 10]	[3, 6, 9]
S_3	[1, 3, 5]	[0, 3, 7]	[0, 2.33, 5]	[3, 7, 9]
S_4	[1, 3, 5]	[1, 3.67, 7]	[3, 7, 9]	[1, 4.33, 7]
S_5	[8, 9, 10]	[8, 9, 10]	[3, 7.67, 10]	[6, 8.67, 10]

Express the fuzzy triangular numbers into $[0, \frac{\pi}{2}]$ form.

Table 7: Fuzzy-decision matrix and fuzzy weights of five candidates

Performance	C_1	C_2	C_3	C_4
Weight	[0.667, 0.833, 0.933]	[0.4, 0.6, 0.767]	[0.4, 0.6, 0.767]	[0.667, 0.833, 0.933]
S_1	[0.942, 1.256, 1.413]	[0.471, 1.204,1.57]	[0.471, 1.151, 1.57]	[0.942, 1.3612, 1.57]
S_2	[0.157, 0.471, 0.785]	[0.942, 1.256, 1.413]	[0.471, 1.204, 1.57]	[0.471, 0.942, 1.413]
S_3	[0.3140,0.5233,1.57]	[0, 0.471, 1.099]	[0, 0.3658, 0.7850]	[0.471, 1.099, 1.413]
S_4	[0.157, 0.471, 0.785]	[0.157, 0.576, 1.099]	[0.471, 1.099, 1.413]	[0.157, 0.68, 1.099]
S_5	[1.256, 1.413, 1.57]	[1.256, 1.413, 1.57]	[0.471, 1.204, 1.57]	[0.942, 1.361, 1.57]

Table 8: Normalized fuzzy decision matrix using $sin(x^*)$ transform:

Performance	C_1	C_2	C_3	C_4
Weight	[0.667, 0.833, 0.933]	[0.4, 0.6, 0.767]	[0.4, 0.6, 0.767]	[0.667, 0.833, 0.933]
S_1	[0.8087,0.9509,0.9876]	[0.4538,0.9335, 1]	[0.4538,0.9131,1]	[0.8087,0.9781, 1]
S_2	[0.1564,0.4538,0.7068]	[0.8087,0.9509,0.9876]	[0.4538,0.9335,1]	[0.4538,0.8087,0.9876]
S_3	[0.3089,0.4998, 1]	[0, 0.4538, 0.8908]	[0, 0.3577, 0.7068]	[0.4538,0.8908,0.9876]
S_4	[0.1564,0.4538,0.7068]	[0.1564,0.5448,0.8908]	[0.4538,0.8908,0.9876]	[0.1564,0.6286,0.8908]
S_5	[0.9509, 0.9876, 1]	[0.9509, 0.9876, 1]	[0.4538, 0.9335, 1]	[0.8087, 0.9781, 1]

Table 9: Weighted normalized fuzzy-decision matrix

Performance	C_1	C_2	C_3	C_4
S_1	[0.5394,0.7921,0.9214]	[0.1815,0.5601,0.767]	[0.1815,0.5479,0.7670]	[0.5394,0.8148,0.933]
S_2	[0.1043,0.378,0.6595]	[0.3235,0.5705,0.7575]	[0.1815,0.5601,0.767]	[0.3027,0.6737,0.9214]
S_3	[0.2060,0.4163,0.9330]	[0, 0.2723,0.6832]	[0,0.2146,0.5421]	[0.3027,0.742,0.9214]
S_4	[0.1043,0.378,0.6595]	[0.0625,0.3269,0.6832]	[0.1815,0.5345,0.7575]	[0.1043,0.5237,0.8317]
S_5	[0.6342,0.8227,0.933]	[0.3803,0.5925,0.7670]	[0.1815,0.5601,0.767]	[0.5394,0.8148,0.933]

The fuzzy positive ideal solution (FPIS)

$$S^* = [(1,1,1),(1,1,1),(1,1,1),(1,1,1),(1,1,1)]$$

The fuzzy negative ideal solution (FNIS)

$$S^- = [(0,0,0),(0,0,0),(0,0,0),(0,0,0),(0,0,0)]$$

Table 10: Distances between S_i (i=1,2,..,5) and S^* with respect to each criteria

	C_1	C_2	C_3	C_4
$d(S_1,S^*)$	0.2806	0.5531	0.5564	0.2892
$d(S_2,S^*)$	0.5703	0.4834	0.5531	0.4468
$d(S_3,S^*)$	0.5703	0.7371	0.7803	0.4317
$d(S_4,S^*)$	0.5703	0.6910	0.5614	0.5938
$d(S_5,S^*)$	0.2885	0.4488	0.5531	0.2892

Table 11: Distances between S_i (i=1,2,..,5) and S^- with respect to each criteria

	C_1	C_2	C_3	C_4
$d(S_1,S^-)$	0.7737	0.5583	0.5542	0.7800
$d(S_2,S^-)$	0.6017	0.5785	0.5583	0.6818
$d(S_3,S^-)$	0.6017	0.4246	0.3366	0.7050
$d(S_4,S^-)$	0.6017	0.4388	0.5454	0.5703
$d(S_5,S^-)$	0.8053	0.6011	0.5583	0.7800

Table 12: Computation of $d_i^*, d_i^- and CC_i$ using modified TOPSIS method.

Supplier	d_i^*	d_i^-	$d_i^* + d_i^-$	CC_i	Rank
S_1	1.6793	2.6662	4.3455	0.6136	2
S_2	2.0535	2.4202	4.4738	0.5410	3
S_3	2.5193	2.0680	4.5873	0.4508	5
S_4	2.4164	2.1562	4.5726	0.4715	4
S_5	1.5296	2.7447	4.2743	0.6421	1

The ranking order of suppliers is evaluated by using fuzzy TOPSIS method Chen[3]

Table 13: Computation of $d_i^*, d_i^- and CC_i$ using TOPSIS method.

Supplier	d_i^*	d_i^-	$d_i^* + d_i^-$	CC_i	Rank
S_1	2.0006	2.4528	4.4534	0.5508	2
S_2	2.3769	2.1690	4.5460	0.4771	3
S_3	2.8151	1.7313	4.5464	0.3808	5
S_4	2.7324	1.8072	4.5396	0.3981	4
S_5	1.7979	2.5661	4.3640	0.5880	1

Table 14: Computation of cost by using graded mean integration (using equation 1 and Table 7):

Supplier	Fuzzy numbers of cost	Cost (c_i)
S_1	[6, 8, 9]	7.8
S_2	[1, 3, 5]	3
S_3	[1, 3, 5]	3
S_4	[1, 3, 5]	3
S_5	[8, 9, 10]	9

V. RESULTS AND DISCUSSION

Interpretation of CC_i
If $CC_i \in [0, 0.2)$ then do not recommend any supplier.
If $CC_i \in [0.2, 0.4)$ then recommend the supplier with high risk.
If $CC_i \in [0.4, 0.6)$ then recommend the supplier with low risk.
If $CC_i \in [0.6, 0.8)$ then approve the supplier.
If $CC_i \in [0.8, 1]$ then approve and prefer the supplier.

Ranks are assigned to supplier according to descending order. The closeness coefficient of supplier was $S_5>S_1>S_2>S_4>S_3$. Recommended supplier are S_3, S_2 and S_4 with low risk and approved suppliers are S_1 and S_5.

Once we decided the suppliers then next step is how much quantity to purchase from the selected suppliers. Each supplier had some restrictions. Considering these restrictions as a constraint stated in section 3, optimize the closeness coefficient using usual LPP.
5^{th} constraints are obtained by using equation (6) and cost of Table 14. The LPP formulation is given in Appendix.

This problem solved by using LINGO software to get optimal order quantity as $x_1=769$, $x_2=676$, $x_3=0$, $x_4=0$ and $x_5=555$.

Ranking order of suppliers was $\mathbf{S_5>S_1>S_2>S_4>S_3}$ by using the modified TOPSIS method and Ranking order of suppliers was $\mathbf{S_5>S_1>S_2>S_4>S_3}$ by using the fuzzy TOPSIS method Chen [2000]. Fuzzy modified TOPSIS and Fuzzy TOPSIS gave us same ranking but minute examination of data revealed that modified TOPSIS gave us better ranking of suppliers as compared to TOPSIS method.

We applied the $sin(x^*)$ normalization transform. It gave us the better ranking as compared to usual normalization transform. The $sin(x^*)$ normalization transform is 13.27% more variable transform as compared to transform used by Chen [2000]. That is reflected in the computation of closeness coefficient. Using modified TOPSIS, suppliers are selected. But we don't get idea about what will be the order quantity from each selected suppliers. LPP method is used to maximize the closeness coefficient subject to constraints. LINGO software is used to solve the LPP. It gave us the order quantity form each suppliers, $x_1=769$, $x_2=676$, $x_3=0$, $x_4=0$ and $x_5=555$. Using this methodology first we got the closeness coefficient. Using closeness coefficient we ranked the suppliers. Then we obtained the optimal solution. Throughout this methodology we used the $sin(x^*)$, $x^* \in \left[0, \frac{\pi}{2}\right]$ transform because $sin(x^*)$ transform captures the most of variation as compared to usual transform that used in fuzzy TOPSIS method (Appendix). This huge variation (13.27%) leads to isolate the ranking of supplier in better way. It means while evaluating closeness coefficient, we get better ranking.

VI. CONCLUSION

This study presents a multi-criteria group decision making for selection of suppliers using the fuzzy modified TOPSIS method. Fuzzy modified TOPSIS method is suitable to deal with the uncertainty. In this process, the ratings of each supplier with respect to each criterion are given as the linguistic variables and the weights of each criterion are also linguistic. $sin(x^*)$, $x^* \in \left[0, \frac{\pi}{2}\right]$ transform is used for normalization and the closeness coefficient is evaluated. Using LPP, we get an optimal order quantity from the selected suppliers. This approach gives us very important aspect of the optimal order quantity after selecting the suppliers. Modified TOPSIS method is fusion of Fuzzy TOPSIS method and linear programming problem (LPP) method along with more variable transform $sin(x^*)$. Thumb of rule is that more the variable transforms better the closeness coefficient. $sin(x^*)$ transform is more variable as compared to transform used for normalization in Fuzzy TOPSIS method. Hence we get better closeness coefficient. Modified TOPSIS method is better than Fuzzy TOPSIS method.

In real cases, many input data are not known precisely for decision making. In this model, imprecise nature of data and varying importance of the quantitative and the qualitative criteria are considered. In real cases, the proposed model would be beneficial to DM for finding out the appropriate order quantity to each selected supplier and allows supply chain manager(s) to manage supply chain performance on cost, quality, on time delivery and long term relationship. The modified TOPSIS method not only ranks the supplier but also it gives us the idea about how much to order from the selected supplier. The modified TOPSIS method for the supplier selection can be applied in education, management and Industry for decision making. This method can be improved by using the other more variable transform to solve the supplier selection problem more efficiently and developing group decision support system in the fuzzy environment can be considered as a topic of future research.

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APPENDIX

TOPSIS Method:-

The transform used in TOPSIS method is $g(x) = \frac{x}{10}$.

The density of uniform distribution is $f(x) = \begin{cases} \frac{1}{10} & for\ 0 \le x \le 10 \\ 0 & otherwise \end{cases}$

And the variance of this transform is $\left(g(x)\right) = 0.0833$.

Modified TOPSIS method:-

The transform used in TOPSIS method is $g(x) = \sin(x)$.

The density of uniform distribution is $f(x) = \begin{cases} \frac{2}{\pi} & for\ 0 \le x \le \frac{\pi}{2} \\ 0 & otherwise \end{cases}$

And the variance of this transform is $\left(g(x)\right) = 0.094358$.

The LPP formulation of closeness coefficient is as follows.

Max $Z = 0.6136\ x_1 + 0.5410\ x_2 + 0.4508\ x_3 + 0.4715\ x_4 + 0.6421\ x_5$;
Subject to the constraint
$x_1 + x_2 + x_3 + x_4 + x_5 = 2000$
$x_1 \le 800$
$x_2 \le 700$
$x_3 \le 750$
$x_4 \le 850$
$x_5 \le 650$
$7.8\ x_1 \le 6{,}000$
$3\ x_2 \le 5{,}000$
$3\ x_3 \le 6{,}000$
$3\ x_4 \le 6{,}000$;
$9\ x_5 \le 5{,}000$;
all variables are nonnegative and integer.

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