Contribution to the Evaluation of Steel Structures Resistance to Lateral Displacement's

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***Abstract*—The choice of the global analysis method of a steel structure is essentially related to its sensitivity to second order effects. This sensitivity depends on the structural strength to the lateral displacement's. The classification of structure as "flexible structure" or "rigid structure" allows choosing the required method of analysis for the latter. From a regulatory point of view, a structure can be classified as rigid, if the ratio of the value of elastic critical load for the instability into the sway mode to the value of design vertical load is greater than ten. In practice, the calculation of the elastic total vertical load is not easy. For this reason, studies have been made in this field and have accomplished the proposal of simple expressions computing as an alternative to the direct determination of the critical elastic load of the structure. The main objective of this work is to explore these alternative methods in order to extend the study in this field and to evaluate their robustness and the results of its application on different types of structures.**

***Index Terms*— Buckling, Critical load, Global analysis method, Instability, Second order effects**

I. Introduction

Internal forces in steel structures can generally be determined using one of the two following analytical methods:

- First order global analysis method, referring to the initial geometry of the structure.
- Second order global analysis method, taking into consideration the influence of structure's deformation.

The choice of the global analysis method is essentially related to the sensitivity of the structure to the second order effects under a given load. This sensitivity depends on structural strength to the lateral displacement.

All structures must have sufficient rigidity in order to limit the lateral deformation. In unbraced structures some beam to column connections must be moment resisting in order to transmit horizontal forces to the foundations and to provide frame stability. The classification of this type of structures into:

- flexible structure;
- rigid structure;

allows choosing the global analysis method required for this structure.

A structure can be classified as rigid structure if its response to the horizontal loads in the plan is sufficiently rigid that we can with acceptable accuracy, neglect the additional stresses generated by taking into account the horizontal movement of its nodes. Otherwise, the structure must be classified as flexible and the effects of horizontal displacement of its nodes must be taken into account in its calculation.

In CCM97 Regulations [1] and Eurocode 3 [2], the effects of the geometry's deformation of the structure is estimated by calculating the α_{cr} factor (usually called elastic critical multiplier), defined as:

$$\alpha_{cr} = F_{cr} / F_{Ed} \tag{1}$$

F_{cr} : is the elastic critical load of instability in a global mode, based on the initial elastic rigidities,

F_{Ed} : is the calculation's load applied to the structure.

Provided the framework is sufficiently rigid, the second order effects may be ignored and a first order analysis can be performed. From a regulatory perspective (CCM97, Eurocode 3), it is the case if:

- for an elastic analysis: $\alpha_{cr} \geq 10$,
- for a plastic analysis: $\alpha_{cr} \geq 15$.

II. Theory and Evaluation Methods of the Elastic Critical Load

The elastic critical load for calculating the α_{cr} factor can be determined by several processes:

- Analytical evaluation.
- Numerical calculation.
- Approximate methods.

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A. Analytical evaluation

Analytical methods consist of a direct solution of the differential equilibrium equations, in order to satisfy the boundary conditions [3,4,5,6,7]. These methods have many difficulties and can only be carried out for simple buckling problems, for structures of which the freedom degrees number is low [.

B. Numerical calculation

The critical load value that can likely buckle the compressive elements of the structure can be determined using numerical methods. This approach consists of resolving the problem of the eigen values, in a manner similar to the resolution of a modal analysis [8]. The difference from the modal case lies in the type of the matrix that is taken into consideration.

For buckling analysis, the calculation is done taking into account the classical elasticity matrix of the structure [K] and integrating the geometrical stiffness matrix [K_g]. The geometrical stiffness matrix contains terms that are function of stress state value in the elements.

By integrating these two matrices, the equation to be solved is the following:

$$\lfloor\lfloor K\rfloor + \lfloor K_g\rfloor\rfloor\{u\} = \{F\} \tag{2}$$

With:
$\{u\}$: displacements vector of the structure,
$\{F\}$: vector of external forces.
Integrating in the calculation of the geometrical stiffness matrix, the stiffness terms of the two matrix may be in opposition. Taking the example of a bending beam, this beam would have a lower stiffness if it is initially compressed and higher if it was tense. This example highlights the philosophy of generalized buckling method where we search the α_{cr} coefficient, which; multiplied by the load, cancels the structure's rigidity. So we are led to solve the following equation:

$$\det\lfloor\lfloor K\rfloor - \alpha_{cr}\lfloor K_g\rfloor\rfloor = \{0\} \tag{3}$$

Thus, in this analysis it is to determine the critical amplification coefficient α_{cr} which applied to the initial loading of the structure leads to buckling.

C. Approximate methods

The approximate methods have been proposed as an alternative to the direct determination of the critical load of the structure [9,10,11,12].

Among those methods we can mention that of *Wood* [12] which is also known as the method of *Merchant - Rankine - Wood*. This method was developed in order to estimate the buckling length of compressed elements of plane frames, in the case of buckling in a sway mode rather than of buckling in a non-sway mode. According to this method the ratio of the buckling length (L_e) and the real length of a column is determined based on the distribution factors η_1 and η_2 obtained by considering the stiffness of the adjacent bars to the nodes of this column. For each column of the structure, we first calculate the critical load of the elastic buckling, using the expression:

$$N_{cr} = \pi^2 EI / L_e^2 \tag{4}$$

The α_{cr} factor is obtained by comparing the critical load to the value of the computational load N_{Ed} applied to this column:

$$\alpha_{cr} = N_{cr} / N_{Ed} \tag{5}$$

This process must be repeated for all columns of the structure in order to find the smallest α_{cr} factor which will be that of the structure.
Another approximate method which was adopted by CCM97 Regulations and Eurocode 3 is that of *Horne* [9,10,11].
Horne method is applicable to plane frame and for one storey frame with a low sleeper's slope (≤26°). These frame are not braced and their sleepers should be loaded with low axial loads. This method consists of calculating for each level the α_{cr} factor by the expression:

$$\alpha_{cr} = \left(\frac{H_{Ed}}{V_{Ed}} \times \frac{h_i}{\delta_{H,Ed}}\right) \tag{6}$$

H_{Ed} is the total horizontal reaction at the bottom of the storey;
V_{Ed} is the total vertical reaction at the bottom of the storey;
$\delta_{H,Ed}$ is the horizontal displacement at the top of the storey compared to its lower part;
h_i is the height of the storey.
In this case also, the smallest α_{cr} factor will be that of the structure.

III. Comparative Study on the Elastic Critical Multiplier

The present study consists of preceding a numerical experiment in order to compare the elastic critical multiplier α_{cr} through two approaches. The first approach is based on a numerical analysis via a numerical program which is based on solving a problem of eigen values considering the matrix of classical stiffness of a beam element and associating it with the geometrical stiffness matrix . The second approach is to use the approximate method of *Horne* since it was adopted by the regulation. This method is applicable when the structure responds at certain topologies criteria, frequently satisfied for usual structures of building.

The benefit of this comparative study is to evaluate the robustness of the regulatory approximation method and the results of its application on different types of structures.

For this study, we consider several frames composed of HEB260 columns, and IPE400 beams for sleepers, subjected to different cases of vertical and horizontal loads. For these structures, we take different boundary conditions at base supports (recessed, articulated) and two storey heights (5m and 10m). For frames with a single level, we consider several slope values of the sleeper (0°, 10°, 20°, 25°, 30° and 40°).

A. Analysis of one storey frames without inclination of the sleeper

In this case, the structures under study corresponds to one storey frames without inclination of the beam recessed at base supports and then articulated at base supports. In a first step, it was taken a column height of h = 5m. The results of both approaches to calculate α_{cr} factor are shown in the following figures:

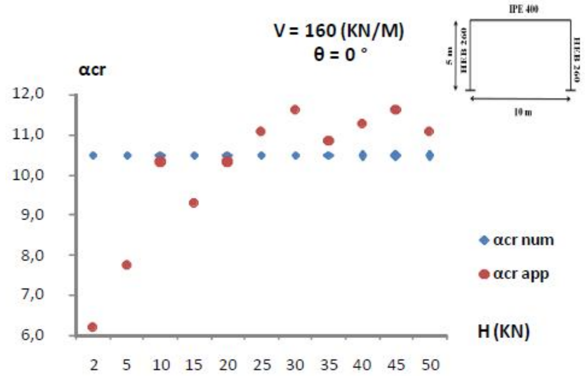

Fig 1: Results of the case of a one storey plane frame recessed at base supports with h = 5m

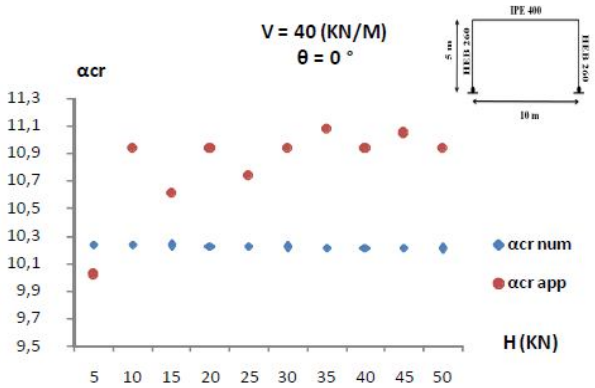

Fig 2: Results of the case of a one storey plane frame articulated at base supports with h = 5m

The value of the vertical load was selected to obtain numerically a value of the α_{cr} factor near to 10.

In a second step, a column height of h = 10m has been considered with the same boundary conditions considered in the first step. The results of both approaches for calculating the α_{cr} factor are shown in the following figures:

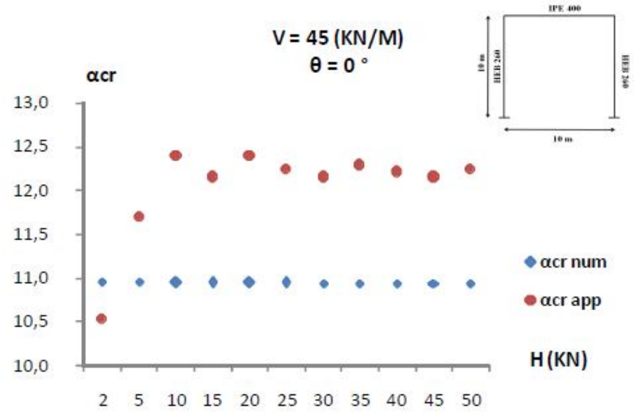

Fig 3: Results of the case of a one storey plane frame recessed at base supports with h = 10m

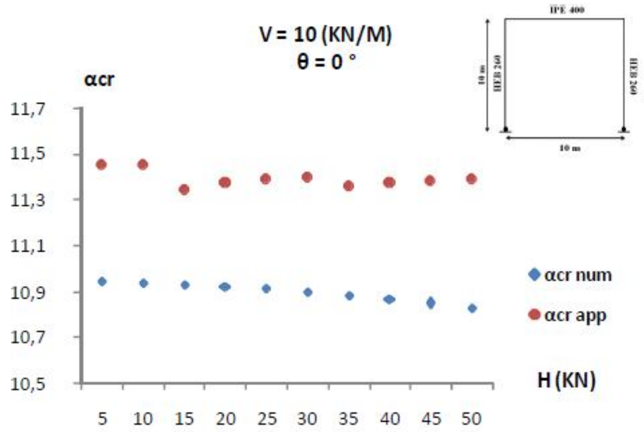

Fig 4: Results of the case of a one storey plane frame articulated at base supports with h = 10m

In each studied case, the value of the vertical load was kept constant by varying that of the horizontal load. The obtained results show that the numerical values of the α_{cr} factor remain substantially constant for each studied case which is not the case for values given by the regulatory approach. The most significant differences were obtained in the case of a right frame recessed at base supports for h = 5m. It was also found that the lowest values given by the regulatory approach were obtained for low values of the horizontal load.

B. Analysis of one storey frames with inclination of the sleeper

The studied frames in the previous step were also analyzed in regard to various slope values of the sleeper (10°, 20°, 25°, 30° and 40°).

In the case of a recessed supports frame with a sleeper of a slope (θ = 20°) and columns of 5m height, results of the two approaches in order to calculate the α_{cr} factor are shown in following figure:

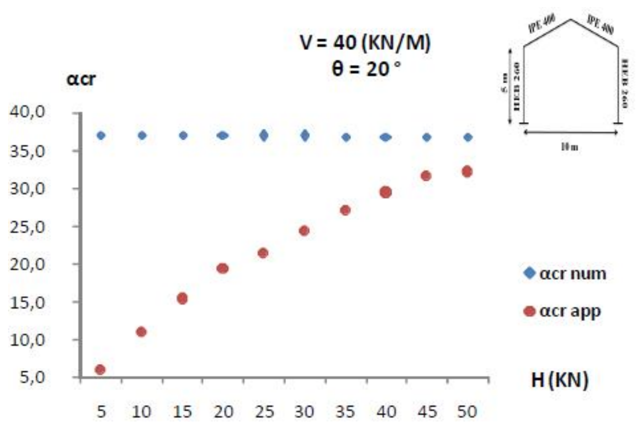

Fig 5: Case of a one storey frame with inclination of the beam recessed at base supports with h = 5m

The results of other studied cases are summarized in the tables below.

Table 1: Results of the study of structures with inclination of the beam: case with h = 5m

	Slope of the sleeper	H = 5 kN		H = 50 kN	
		$\alpha_{cr\ num}$	$\alpha_{cr\ app}$	$\alpha_{cr\ num}$	$\alpha_{cr\ app}$
Recessed	θ = 0°; (V = 160 kN/m)	10,49	7,73	10,46	11,05
	θ = 10°; (V = 80 kN/m)	20,20	2,53	20,10	14,20
	θ = 20°; (V = 40 kN/m)	37,09	5,93	36,74	32,16
	θ = 25°; (V = 35 kN/m)	40,01	7,00	39,60	37,94
	θ = 30°; (V = 30 kN/m)	43,42	8,35	42,92	45,27
	θ = 40°; (V = 20 kN/m)	53,37	6,00	52,49	78,59
Articulated	θ = 0°; (V = 40 kN/m)	10,23	10,02	10,20	10,93
	θ = 10°; (V = 35 kN/m)	11,39	6,09	11,35	12,77
	θ = 20°; (V = 37 kN/m)	10,11	4,16	10,07	11,88
	θ = 25°; (V = 35 kN/m)	10,14	3,95	10,10	12,47
	θ = 30°; (V = 32 kN/m)	10,38	4,08	10,33	13,62
	θ = 40°; (V = 27 kN/m)	10,25	4,14	10,20	15,53

Table 2: Results of the study of structures with inclination of the beam: case with h = 10m

	Slope of the sleeper	H = 5 kN		H = 50 kN	
		$\alpha_{cr\ num}$	$\alpha_{cr\ app}$	$\alpha_{cr\ num}$	$\alpha_{cr\ app}$
Recessed	θ = 0°; (V = 45 kN/m)	10,95	11,70	10,93	12,24
	θ = 10°; (V = 45 kN/m)	10,74	5,64	10,72	11,64
	θ = 20°; (V = 45 kN/m)	10,15	3,78	10,13	10,85
	θ = 25°; (V = 40 kN/m)	10,88	3,88	10,86	11,90
	θ = 30°; (V = 40 kN/m)	10,30	3,22	10,27	11,20
	θ = 40°; (V = 35 kN/m)	10,06	2,82	10,03	11,46
Articulated	θ = 0°; (V = 10 kN/m)	10,94	11,45	10,82	11,38
	θ = 10°; (V = 10 kN/m)	10,75	11,32	10,60	12,09
	θ = 20°; (V = 10 kN/m)	10,19	10,69	10,00	12,40
	θ = 25°; (V = 9 kN/m)	10,71	11,49	10,45	13,50
	θ = 30°; (V = 8 kN/m)	11,25	12,12	10,88	14,74
	θ = 40°; (V = 7 kN/m)	10,91	12,01	10,42	15,73

The observations made for a frame without inclination of the beam remain valid in this case, we always note that the numerical values of α_{cr} factor remain substantially constant for each studied case which is not the case for the values given by the regulatory approach. The differences observed between the two approaches for low values of the horizontal load become even more pronounced with the increase in the slope of the sleeper.

C. Analysis of multi storey frames

In order to complete this numerical experiment, we have also analyzed structures of seven levels considering storey heights of 5m then 10m. Each structure was studied in both cases recessed and articulated at base supports.
The figure below shows the results for the structure recessed at base supports with storey heights of 10m.

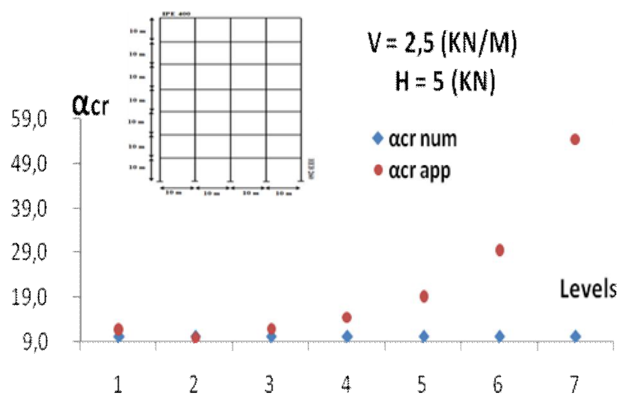

Fig 6: Results for structure with 7 levels recessed at base supports with h = 10m

The results of the other studied cases are given in the table below.

Table 3: Results of the study of multi storey frames

	Levels	H=5 kN					
		Recessed			Articulated		
		V(kN/m)	$\alpha_{cr\ num}$	$\alpha_{cr\ app}$	V(kN/m)	$\alpha_{cr\ num}$	$\alpha_{cr\ app}$
h = 5 m	1	12	10,19	12,57	3	10,55	10,29
	2	12	10,19	8,97	3	10,55	24,46
	3	12	10,19	10,47	3	10,55	32,61
	4	12	10,19	15,71	3	10,55	39,13
	5	12	10,19	15,71	3	10,55	48,92
	6	12	10,19	31,42	3	10,55	65,22
	7	12	10,19	31,42	3	10,55	195,69
h = 10 m	1	2,5	10,32	11,56	1	4,54	4,68
	2	2,5	10,32	10,04	1	4,54	13,21
	3	2,5	10,32	11,92	1	4,54	17,61
	4	2,5	10,32	14,68	1	4,54	22,36
	5	2,5	10,32	19,08	1	4,54	29,07
	6	2,5	10,32	29,36	1	4,54	44,72
	7	2,5	10,32	54,52	1	4,54	83,05

These results show that the numerical values are identical for each level. The values given by the approximate method are very different from one level to another; however the lowest given value is close to that found numerically for the same studied case.

IV. CONCLUSION

In this study, we have analyzed several plane frames vertically as well as horizontally loaded. The differences between the structures were achieved by varying:

1. the boundary conditions at the base of the structure (recessed, articulated);
2. the level heights (h = 5m, h = 10m);
3. the number of levels;
4. the value of the sleeper's slope θ (0°, 10°, 20°, 25°, 30° and 40°).

The analysis was made considering a numerical approach as well as the regulatory approach based on an approximate method.

For each studied case, the vertical load value was kept constant by taking several values of the horizontal load. The values of the elastic critical multiplier α_{cr} given by the numerical approach were practically the same as the horizontal load varies; by contrast those given by the approximate method vary along the horizontal load.

The importance of the variation in the approximate values depends on the studied structure. In the case of one storey frame, this variation is greater when:

1. the structure is recessed at base supports;
2. the height of the columns is smaller (h = 5 m);
3. the slope of the sleeper increases.

The values of the elastic critical multiplier α_{cr} given by the approximate method for multi storey structures are close to those found numerically.

This study should be continued and extended to other aspects that may influence the stability of frameworks. Among these aspects we can mention:

1. Taking account of different relationships between the rigidities of beams and columns of the structure.
2. The consideration of structures at several levels with different storey heights in the same structure.
3. The consideration of structures with different beam lengths in the same structure.
4. Consideration of the semi- rigid nodes "beam-column".

The study of these aspects will allow suggesting corrections for the approximate method considered in this study leading to satisfactory results.

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