CUMULATIVE SUM, CUMULATIVE SCORE AND NONPARAMETRIC CUSUM CONTROL CHARTS FOR DETECTING MEAN/MEDIAN CHANGE

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Abstract— **Various control charts are already developed for the problem of detecting any shift in the mean/median of a sequence of observations from a specified control value taken from some process. Some of them for detecting small shift(s) from the target values are Cumulative Sum (CUSUM), Cumulative Score (CuScore) and recently developed nonparametric charts. Various authors have already worked on CUSUM control charts and found suitable results. Another chart is Cuscore chart, which is also suitable for this situation. Another test is nonparametricCUSUM charts based on nonparametric test statistic.**

One of the efficient procedure is the Cumulative Sum Control chart (CSCC) originally introduced by Page [1]. The properties of CSCC procedure are usually derived under the assumption that the observations are independent and identically distributed normal random variables, another control chart based on Cumulative Scores developed by Munford [2]. The ARL of this scheme are simpler to compute than those of CSCC. Recently a new control chart is developed based on the nonparametric chart which is preferable from the robustness point of view. Few of the workers on nonparametric CSCC are Parent [3], Reynolds [4, 5], Mcgilchrist and Woodyer [6], Bakir and Reynolds [7] etc.

In this paper we want to study the performance of CSCC, CuScore and Nonparametric CUSUM control Chart in detecting the mean/median shift from the specified (target) values. For this purpose we computed the ARL by the simulation method for both in control and under control situations of the process. Results obtained are displayed in various tables using different shift parameters and under different distributions. Results are also shown in graph for easy visual comparison.

Index Terms— **CSCC, CuScore Chart, Nonparametric chart, ARL, Simulation.**

I. INTRODUCTION

Statistical Process Control (SPC) has been widely used to monitor various industrial processes. Most of the research works in SPC focuses on charting techniques. Some control charts such as Shewhart charts, CUSUM charts and EWMA charts are useful tool in detecting the shifts in the process mean and/or deviation. These charts are used to monitor product quality and detect special events that may be indicators of out-of-control situations. These charts are based on the basic assumption that a process being monitored will be produce measurements that are independent and identically distributed over time when only inherent sources of variability are present in the system.

It is well known that CUSUM procedures give tighter process control than the classical quality control schemes, such as Shewhart schemes. Another effective alternative to Shewhart Control chart is Exponentially Weighted Moving Average (EWMA) chart. The above two alternatives are especially effective for detecting relatively small shifts. The properties of these procedures have been widely discussed in the statistical literature; see, for example, Page [1], Evan and Kemp [8], Bissell [9], Roberts [10, 11], Lucas and Saccucci [12].

Shewhart type control charts only use the last sample to monitor the process. These charts have no memory: previous observations do not influence the probability of future out-of-control signals. Again, Shewhart Charts are insensitive to small shifts. They are very effective if the magnitude of the shift is 1.5 sigma or 2 sigma or larger. For smaller shifts in the process mean and/or deviation, the CUSUM and EWMA are good alternative to the Shewhart chart.

In EWMA type control charts the process is monitored using a weighted mean of all previous observations. The weight attached to recent observation is high compared to the weights of older observations. The weight decline exponentially as the observation gets older and older. The parameter λ determines the memory of the EWMA chart.

The CUSUM chart, which was originally introduced by Page [1], uses an unweighted sum of all previous observations. This chart has a rather long memory.

Now occasions may occur when some specific kind of deviation other than a change in mean is feared as a likely possibility. To cope with this kind of problem, Shewhart chart may be used as one of many virtues of the Shewhart chart is that it is a direct plot of actual data and so can expose

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types of deviations from statistical ability, of a totally unexpected kind. However, when we can identify in advance a kind of departure specifically feared, then a more sensitive detection statistic, known as Cumulative score statistic, proposed by Box and Jenkins [13] in an unpublished report can be used. Munford [2] developed process control schemes using CuScore statistic.

In this article, a comparative discussion is made to study the performance of three control charts, viz., the parametric Tabular CUSUM, a nonparametric CUSUM chart based on sign statistic, and a CuScore control chart. For this purpose we computed the ARL of the three processes by the simulation method for both in control and under control situations. Results obtained are displayed in various tables using different shift parameters and under different distributions. Results are also shown in graph for easy visual comparison and accordingly, discussion and conclusions are made.

II. DESCRIPTION OF THE CONTROL CHARTS

In this section we would briefly discuss the various parametric as well as nonparametric control charts that we are considering in this paper for comparison purpose.

A. CUMULATIVE SUM CONTROL CHART (CSCC)

The Cumulative Sum (CUSUM) Control charts were, initially proposed by Page [1] being studied by many authors.

The tabular or algorithmic CUSUM is defined as

$$C_i^+ = \max[0, x_i - (\mu_0 + k) + C^+_{i-1}] \quad \ldots (2.1)$$

$$C_i^- = \max[0, (\mu_0 - k) - x_i + C^-_{i-1}] \quad \ldots (2.2)$$

where the starting values are $C_0^+ = C_0^- = 0$. K is usually called the reference value or allowable value or slack value. K is often chosen between the target μ_0 and the out-of-control value of the mean μ_1 that we are interested in detecting quickly. Thus if the shift is expressed in standard deviation units as $\mu_1 = \mu_0 + \delta\sigma$

or $\delta\sigma = \mu_1 - \mu_0$ or $\delta = \frac{|\mu_1 - \mu_0|}{\sigma}$ then

$$K = \tfrac{1}{2}\delta\sigma = \frac{|\mu_1 - \mu_0|}{2}$$

.

A reasonable value for H is five times the process standard deviation σ. Proper selection of the two parameters, viz., reference Value K and decision Interval H is quite important as it has substantial impact on the performance of the CUSUM. Note that $C_i^+ and C_i^-$ accumulate deviation from the target value μ_0 that are greater than k, with both quantities reset to zero on becoming negative. If either $C_i^+ or C_i^-$ exceed the decision interval H, the process is considered to be out-of-control.

CUSUM for rational subgroups

The tabular CUSUM can be easily extended to the case of averages of rational subgroups (n>1). One have to simply replace X_i by $\overline{X}_i$ (the sample or subgroup average) in the above formulas (2.1) and (2.2), and replace σ with i.e.

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} \qquad Y_i = \frac{\overline{X}_i - \mu_0}{\sigma / \sqrt{n}}$$

Then the standardized two-sided CUSUM is

$$C_i^+ = \max[0, Y_i - k + C^+_{i-1}] \quad \ldots (2.3)$$

$$C_i^+ = \max[0, -k - Y_i + C^-_{i-1}] \quad \ldots (2.4)$$

With Shewhart charts, the use of averages of rational subgroups substantially improves control chart performance. However, this does not always happen with CUSUM. Only if there is some significant economy of scale or some other valid reason for taking samples of size greater than one be used with the CUSUM.

B. *CONTROL CHART BASED ON CUMULATIVE SCORES*

In the CUSUM scheme (Page[1]) for detecting increases in the mean from its target value, the cumulative sums of the differences between the sample means and some reference value K, $\Sigma(\overline{X}_j - K)$, are plotted against sample number. If the CUSUM becomes negative, the cumulation is restarted, but if it reaches some value H (the decesion interval), then corrective action is indicated. For two-sided control, it is necessary to test also for decrease in the mean, and this is done by operating a second CUSUM with reference value and decision interval –K and –H respectively.

The schemes developed by Munford [2], assign a score of -1, +1 or 0 to the sample means according to whether they are extreme negative, extreme positive or otherwise. In the two- sided case corrective action is indicated when the modulus of the cumulative score reaches some fixed value, which amounts to operating a CuSum on the scores with zero reference value. In the one-sided case, a new decesion rule is developed by Munford [2]. Like the Shewhart scheme, both schemes have the attractive property that the Average Run Length (ARL) can be expressed as a simple function of the tail areas of the quality distribution, and the basic Shewhart scheme is in fact a special case. The advantage of these schemes is that they can be more sensitive to small deviations in the process mean than Shewhart schemes, at the expense of some efficiency for larger deviations.

The two-sided case of CuScore chart due to Munford [2] may be stated as:

Without loss of generality, we may scale the variables so that $\mu_0 = 0$ and $\sigma/\sqrt{n} = 1$. Let k be a positive constant, and define the random variables U1,U2,… by

$$U_j = \begin{cases} -1, \bar{X}_j \le -k \\ 0, -k < \bar{X}_j < k, j = 1, 2, ... \\ 1, k \le \bar{X}_j, \end{cases}$$

Thus we have Sj=Sj-1+Uj , j = 1,2,… where initial value S0=0,

Here, Sj represents the cumulative score of the first j samples. A positive value of Sj would indicate an excess of high sample means amongst the first j samples, a negative of Sj the reverse.

Page [14] has suggested that a plot of Sj against j be recorded, and each point then tested with a V-mask. Munford [2] scheme uses the same plotting positions, but the mask is replaced by fixed boundaries at $S = \pm a$ (where a is a positive integer); corrective action is taken as soon as $|S_j| = a$. The Shewhart chart thus corresponds to the special case a =1.

The motivation for the stopping rule with fixed boundaries is as follows:

When the process is in control $(\mu = 0)$, Sj represents the position of a particle undergoing a one-dimensional symmetric random walk between absorbing barriers. For a suitably chosen value of K the ARL can be made up as large as necessary, for any value of a. As soon as the process goes out of control, the associated random walk develops a drift, and the expected time to reach absorption is much less.

To compute $ARL(\mu)$, first note that U1, U2,… have the common distribution

$$P\{U = -1\} = q,$$
$$P\{U = 0\} = 1 - p - q,$$
$$P\{U = 1\} = p,$$

where $p = p(\mu) = 1 - \Phi(k - \mu)$ …(2.5)

$$q = q(\mu) = 1 - \Phi(k + \mu) \quad \text{…(2.6)}$$

Let $E_i = E_i(\mu)$ be the expected number of further samples required until corrective action is taken, given that the current S value is i , $i = -a, ..., a$. By considering the three possibilities that may arise when the process is next sampled we have

$$E_i = 1 + qE_{i-1} + (1 - p - q)E_i + pE_{i+1},$$
$$i = -a + 1, ..., a - 1 \quad \text{…(2.7)}$$

Solving the difference equation (2.7) and using the conditions $E_{-a} = E_a = 0$ gives

$$\begin{cases} \frac{(a^2 - i^2)}{2p}, p = q \\ \frac{1}{(p-q)}[(a-i) + \frac{2a}{1-(q/p)^{2a}}\{(\frac{q}{p})^{2a} - (\frac{q}{p})^{a+i}\}], p \ne q \\ i = -a, ..., a \end{cases}$$

…(2.8)

The Average Run Length (ARL) has the same value as E0 , So after a little algebra we get

$$ARL(\mu) = \begin{cases} \frac{a^2}{2p}, p = q \\ \frac{a}{(p-q)}\{\frac{2}{1+(q/p)^a} - 1\}, p \ne q \end{cases}$$

….(2.9)

C. SIGN-CUSUM CHART FOR MONITORING THE PROCESS CENTER

The parametric CUSUM chart (Page [1]) for detecting a shift in a normal mean is based on the cumulative sum of differences from target. The Nonparametric CUSUM chart discussed here (Amin, Reynolds, and Bakir, [15]) uses a cumulative sum of sign test statistic SN_i. A one sided chart for detecting positive deviations from the in-control median value signals at the first t for which

$$\sum_{i=1}^{t}(SN_i - k) - \min_{0 \le u \le t}\sum_{i=1}^{u}(SN_i - k) \ge h \quad \text{… (2.10)}$$

where, h>0 and k>0 are parameters of the procedure. A one-sided chart for detecting negative deviations signals at the first t for which

$$\max_{0 \le u \le t}\sum_{i=1}^{u}(SN_i + k) - \sum_{i=1}^{t}(SN_i + k) \ge h \quad \text{… (2.11)}$$

The corresponding two-sided chart signals at the first t for which either of the one-sided charts signals. An alternate and equivalent way to apply the CUSUM chart involves the use of a graphical V-mask scheme(e.g. Van Dobben de Bruyn, [16]).If k and h are non-negative integers then the above one-sided positive procedure is equivalent to a discrete time Markov chain $\{SN_t^*, t = 0, 1, 2, ...\}$, with the state space a subset of{ 0,1,2,……,h}, where $SN_0^* = 0$ and

$SN_t^* = \min\{h, \max\{0, SN_{t-1}^* + \{SN_t - k\}\}$ ... (2.12)

Where the state h is an absorbing state, and absorption corresponds to a signal by the procedure.

The ARL of CUSUM chart using SN_i can thus be determined from the mean absorption times for the state h. Let $\underline{m}' = (m_0, m_1, \ldots, m_{h-1})$, where m_j is the mean absorption time, given that the chain started initially in state j. If the CUSUM chart starts with $SN_0^* = 0$, then the ARL is just m_0. If Q is the hxh matrix of transition probabilities for the nonabsorbing states of the Markov chain, I is the hxh identity matrix, and $\underline{1}$ is an hxl vector with all elements being unity, then it is well known that $\underline{m}$ is given by (I-Q)-1$\underline{1}$. The transition probabilities for the Markov chain can be easily computed since the distribution of SN_i can be obtained from the binomial distribution.

The value of the ARL for the CUSUM chart using SN_i depends on the values of the parameters h and k. One approach to selecting h and k is to choose the parameter values that minimize $L^+(\mu_1)$ subject to maintaining a specified value of $L^+(\mu_0)$, where μ_1 is a value of μ that is considered as a significant shift. The optimal value for k is then approximately $k = \frac{1}{2} E[SN_i / \mu_1],$ (Reynolds [5]). Using this value of k, the value of h should then be chosen to achieve the desired value of $L^+(\mu_0)$.

Approximate optimum values of k for the CUSUM Chart using the Sign statistic when n=10 and $\sigma = 1$ for the uniform, normal, double exponential, Cauchy, and gamma distributions are given by Amin, Reynolds, and Bakir [15]. Except for the Cauchy distribution, the values of k do not differ very much for the various distributions. They have also given the values of $L^+(\mu_0)$ for the CUSUM chart using the Sign statistic for various values of h and k when n=10. They observed that for the Shewhart's charts using SN_i it was necessary to have n of moderate size in order to have a reasonably large value of $L^+(\mu_0)$. For the CUSUM chart however, the value of n can be smaller since the procedure is based on a cumulative sum of statistics from individual samples and h can be chosen large enough to give arbitrarily large values of $L^+(\mu_0)$. The disadvantage of small samples for the CUSUM chart using SN_i is that it is not possible for the procedure to signal after only one sample if n<h+k.

A similar CUSUM chart can be developed for controlling process variability after substituting V_i for SN_i in (2.10) and (2.11). The problem of choosing an appropriate value for k needs a separate investigation.

III. PERFORMANCE COMPARISON OF THE CONTROL CHARTS THROUGH

The performance of a control chart is usually evaluated and compared in terms of the Average Run Length (ARL), which is the expected number of samples required by the procedure to signal that a shift in the process median (mean) μ has occurred. Since the amount of production is proportional to the ARL, it is often used to evaluate control charts in industrial quality control applications. To avoid over-controlling the process, the ARL should be large when the process is operating on target. On the other hand, to minimize the production of lower quality products, the ARL should be small when the process is operating off target (Lucas and Saccucci, [17]). Thus, it is desirable to have a large value of ARL when the process is in control and small value when the process is out of control.

In this section, we compare the performance of the three process control procedures for group of observations, viz, the parametric Tabular CUSUM, CuScore chart and nonparametric Sign-CUSUM chart. The comparisons of the three procedures are carried out by computing the ARL values of each of the procedures by the simulation technique.

We develop computer programs for calculation of each ARL value and 1000 runs are repeated. Normal observations are generated using Box-Muller formula [18] and then necessary values of shift parameters added. The control limits for the control charts, are obtained such that the frequency of the points falling outside the control limits are approximately equal for all the three procedures when the processes are in control. Then the process mean (or median) is shifted by the amount δ and the out -of -control ARL values are recorded. The shifts considered in this study are between δ =.25 to δ =4.

Another point which should be noted is that for all the three procedures, viz, (parametric) Tabular CUSUM, CuScore chart and nonparametric Sign-CUSUM chart, used in this simulation are based on groups of observations of size g. Larger groups sizes allow larger in-control ARL values but can only achieve a minimum out-of-control ARL value of (a larger) g because an entire group must be sampled before a signal. Bakir and Reynolds [7] concluded that the best group size is somewhere between 5 and 10 depending on the desired value of the in-control ARL and the size of the shift.

Again, one important point to be noted here is that, in our present paper observations generated only from Normal distribution is considered as an example, and for observations following other distributions, study has been carried out and supposed to be reported in subsequent articles as this paper is a part of our ongoing research work The parameters of the distribution is chosen such that variance =1. For simplicity, the ARL is calculated under the assumption that the variance of the distribution is known and not estimated.

Thus, in section 3.1 below, we have displayed some of the results obtained through Simulation method.

A. Simulation results

As discussed in detail in section above, using simulation method, we generate normal observations and calculated ARLs for various choices of chart parameters and shifts in the process mean. Thus, the values of ARL's of the three charts viz, the parametric Tabular CUSUM, the nonparametric Sign-CUSUM and the CuScore charts for various degrees of shift in the underlying process average, and magnitudes of the various Chart parameters are presented in the tables below for normal distribution.

Before that, we will first reproduce some given values of ARL performance of the parametric Tabular CUSUM from Montgomery[19] in Table 1 and ARL performance of the Tabular CUSUM and CuScore Control chart in Table 2 from Munford [2] so that we can verify our simulated results with these standard results.

Table 1 : ARL Performance of Tabular CUSUM with k=1/2 and h=4 or h=5

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Shift in Mean (multiple of σ)	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	4.00
h = 4	168	74.2	26.6	13.3	8.38	4.75	3.34	2.62	2.19	1.71
h=5	465	139	38.0	17.0	10.4	5.75	4.01	3.11	2.57	2.01

Munford [2] gives ARL performance of CUSUM with h=2, 5 and k=0.5, 1 and ARL performance of CuScore chart with various a values and corresponding k values, which are reproduced in Table 2 below.

Table 2: ARL Performance of Tabular CUSUM with various h and k and CuScore with

a = 2,3,4,6,8 and k = 2.16, 1.82, 1.54, 2.12, 1.77, 1.48 respectively.[Munford (1980)].

Shift in mean	CUSUM		CUSCORE					
	h=2, k=0.5	h=5,k=1	a=2,k=2.16	a=3,k=1.82	a=4,k=1.54	a=4,k=2.12	a=6,k=1.77	a=8,k=1.48
0.0	463	130	130	130	130	465	465	465
0.5	38	38	44	36	31	82	66	58
1.0	10	10	16	15	14	30	27	26
2.0	4.0	2.7	4.6	5.2	5.9	8.8	10.1	11.5

Now, we tabulate our simulation results i.e the ARL performance of Tabular CUSUM, CuScore and Sign-CUSUM, which we have obtained through our computer programs using simulation technique (Detail is discussed in section 3 above).

The following tables, viz, Table 3(i) and Table 3(ii) gives the ARL performance of the Nonparametric Sign-CUSUM chart for various degrees of shift in the underlying process average, and magnitudes of the various Chart Parameters and subgroup size=1

Table3 (i): ARL performance of Sign-CUSUM with g=10, h=2, 3, 4, 5 and k=1,2,3 Respectively

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	G=10							G=10					
	h=2			h=3				h=4			h=5		
	k=1	k=2	k=3	k=1	k=2	k=3		k=1	k=2	k=3	k=1	k=2	k=3
Shift in mean							Shift in mean						
0.00	5.80	6.98	16.371	6.74	15.74	19.149	0.00	10.434	15.742	46.619	9.86	12.998	39.238
0.25	3.39	3.673	5.998	3.61	5.718	6.8	0.25	4.571	5.718	10.933	3.98	5.146	9.152
0.5	2.53	2.62	3.447	2.53	3.361	3.7	0.5	3.016	3.361	4.717	2.79	3.242	4.405
1.00	2.06	2.068	2.229	2.06	2.227	2.164	1.00	2.21	2.227	2.564	2.16	2.227	2.548
2.00	2.00	2.00	2.003	2.0	2.003	2.003	2.00	2.003	2.003	2.025	2.001	2.003	2.025
3.00	2.00	2.00	2.00	2.0	2.00	2.00	3.00	2.00	2.00	2.00	2.00	2.00	2.00
4.00	2.00	2.00	2.00	2.00	2.00	2.00	4.00	2.00	2.00	2.00	2.00	2.00	2.00

Table3 (ii): ARL performance of Sign-CUSUM with g=10, h=6, 7, 8, 9 and k=1, 2, 3 respectively

Table3 (ii): ARL performance of Sign-CUSUM with g=10, h=6, 7, 8, 9 and k=1, 2, 3 respectively

	G=10							G=10					
	h=6			h=7				h=8			h=9		
	k=1	k=2	k=3	k=1	k=2	k=3		k=1	k=2	k=3	k=1	k=2	k=3
Shift in mean							Shift in mean						
0.00	18.284	39.238	149.214	24.321	95.949	283.138	0.00	32.469	95.949	508.547	42.266	224.135	1073.395
0.25	6.039	9.152	19.802	6.99	13.593	27.073	0.25	7.931	13.593	35.553	8.949	18.635	48.328
0.5	3.708	4.405	6.408	4.026	5.384	7.552	0.5	4.396	5.384	8.647	4.765	6.564	9.737
1.00	2.517	2.548	3.099	2.543	3.065	3.327	1.00	2.901	3.065	3.608	3.031	3.493	3.902
2.00	2.025	2.025	2.215	2.025	2.215	2.217	2.00	2.215	2.215	3.00	2.215	3.00	3.009
3.00	2.00	2.00	2.02	2.00	2.02	2.02	3.00	2.02	2.02	3.00	2.02	3.00	3.00
4.00	2.00	2.00	2.00	2.00	2.002	2.002	4.00	2.002	2.002	3.00	2.002	3.00	3.00

Table 4(a) and Table 4(b) below gives ARL performance of CuScore and Sign-CUSUM chart for various degrees of shift in the underlying process average, and magnitudes of the various Chart Parameters and subgroup size=10. Also, both of these tables gives a comparison of ARL performance of the three procedures viz., the parametric tabular CUSUM chart, the CuScore chart and the non-parametric Sign-CUSUM chart under study for approximate ARL(0) values of (a) 465 and (b) 130

Table4 (a): Comparison of two-sided control schemes

Shift in mean	Tabular CUSUM (k=.5, h=5)	CuScore (a=8, k=2.12)	Sign-CUSUM (h=8, k=2.8)
	g =10		
0.00	465	465.005	470.88
0.25	139	86.566	31.93
0.50	38	27.162	8.23
0.75	17	13.384	4.702
1.00	10	9.447	3.608
1.50	5.75	8.04	3.037
2.00	4.01	8.00	3.00
2.50	3.11	8.00	3.00
3.00	2.57	8.00	3.00
4.00	2.01	8.00	3.00

The ARL values of the three control charts given by table 4(a) have been depicted graphically by Figure1 below.

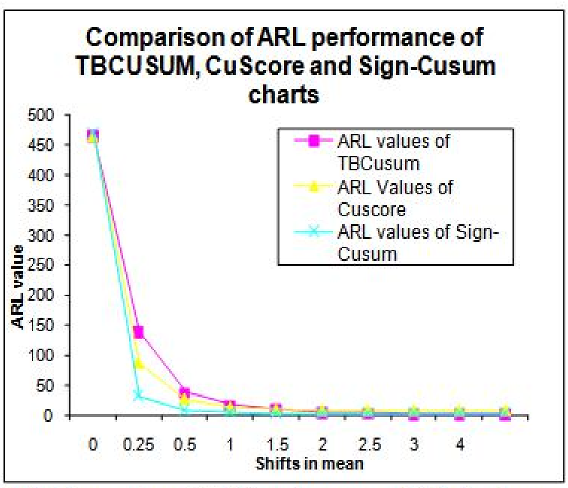

Fig1: Comparison of ARL performance of Tabular Cusum, CuScore and Sign-Cusum chart

Table4 (b): Comparison of two-sided control schemes

Shift in mean	Tabular CUSUM (k=1, h=2)	CuScore (a=2, k=2.16)	Sign-CUSUM (h=6.2, k=2.6)
	g =10		
0.00	121.669	129.168	127.532
0.25	80.364	23.667	16.604
0.50	38.325	7.225	5.752
0.75	18.506	3.426	3.795
1.00	10.161	2.361	3.069
1.50	4.433	2.01	2.508
2.00	2.8	2.00	2.215
2.50	2.1	2.00	
3.00	1.593	2.00	2.02
4.00	1.162	2.00	2.002

Similarly, the ARL values of the three control charts given by table 4(b) have been depicted graphically by Figure 2 below.

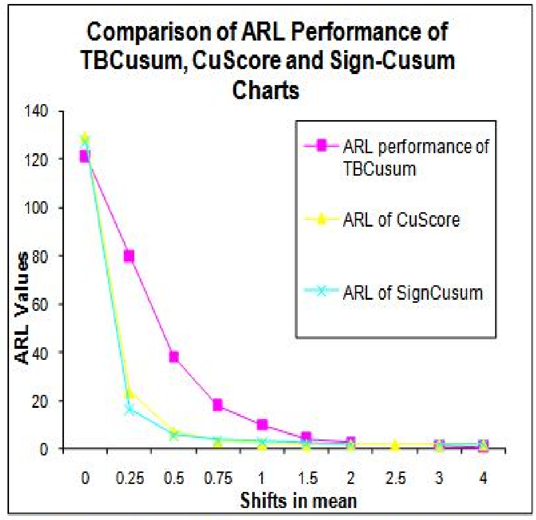

Fig2: Comparison of ARL performance of Tabular Cusum, CuScore and Sign-Cusum chart

IV. CONCLUSION FROM SIMULATION RESULTS

While comparing the ARL performance of the three control chart procedures viz., the parametric Tabular Cusum chart , the CuScore chart and the nonparametric Sign-Cusum chart, from the Table 4(a) and Figure 1 as well as Table 4(b) and Figure 2, it may be concluded as follows:

(1) For the situation given in Table 4(a) and as depicted by Figure 1, the ARL performance of Tabular Cusum and CuScore is almost same and performance of Sign-Cusum is slightly higher for the incontrol mean. But as the shift in mean occurs, the ARL performance of Sign-Cusum seems to be best in this particular situation among the three procedures.

(2) For the situation given in Table 4(b) and as depicted by Figure 2 , for the in-control mean, the ARL performance of the three procedure differ slightly to each other, but as the shifts in mean occurs, the ARL performance of Sign-Cusum seems to be best among the three procedures in this particular situation .

Thus in both the situations we considered to compare the three procedures, indicates that among the three procedures in this particular situation the Nonparametric Sign-Cusum chart seems to give best performance and CuScore chart gives better performance as compared to Tabular Cusum.

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